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Electrical and Computer Engineering

Tables
For
Electrical Engineers

Second Edition

Constants

e	2.718281828	base of natural logarithm
π	3.141592654	pi
ε_o	$8.85418 \times 10^{-12}\ F\ m^{-1} (\approx \frac{10^{-9}}{36p})$	Permittivity of free space
μ_o	$4\pi \times 10^{-7}$ H m^{-1} (exact)	Permeability of free space
c	2.997925×10^{8} ms^{-1}	Velocity of light in free space
k	1.38054×10^{-23} JK^{-1}	Boltzman's constant
h	6.6256×10^{-34} Js	Plank's constant
q	-1.60210×10^{-19} C	electron charge
m	9.1091×10^{-31} kg	electron mass

Conversion Factors

1 neper = 8.686 dB

$\log_2 x = 3.322 \log_{10} x$

$\log_3 x = 2.096 \log_{10} x$

$\log_2 x = 1.443$ 1n x

1n x $= 2.303 \log_{10} x$

In general:

$\log_b x = \log_a x / \log_a b$

Trigonometric and Exponential Formulae

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$

$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$

$\sin x \cos y = \frac{1}{2}(\sin(x + y) + \sin(x - y))$

$\cos x \sin y = \frac{1}{2}(\sin(x + y) - \sin(x - y))$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\frac{x \pm y}{1 \mp xy}$$

$\tan x = -\tan(\pi - x)$

If $y = \sin^{-1} x$ then $\cos y = \sqrt{1 - x^2}$

If $y = \cos^{-1} x$ then $\sin y = \sqrt{1 - x^2}$

$$\cos(\beta \sin \omega t) = J_o(b) + \sum_{n=1}^{\infty} 2J_{2n}(b)\cos 2nwt$$

$$\sin(\beta \sin \omega t) = \sum_{n=0}^{\infty} 2J_{2n+1}(b)\sin(2n+1)wt$$

where $J_n(\beta)$ is the Bessel function of the first kind of order n

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$e^{jx} = \cos x + j \sin x, \qquad j = \sqrt{-1}$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

Series

$$e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^k}{k!} + ...$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} .... \frac{(-1)^k x^{2K+1}}{(2k+1)!} + ...$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .... \frac{(-1)^k x^{2k}}{(2k)!}$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 .... (-1 < x < 1)$$

$$(x+y)^n = x^n + n\,x^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + ...$$ if n integer > 0 or any n if $y^2 < x^2$.

Error Function and Related Functions

$$erf(x) = \frac{1}{\sqrt{2p}} \int_0^x \exp(-\tfrac{1}{2}t^2)\,dt$$

$$erfc(x) = \frac{1}{\sqrt{2p}} \int_x^\infty \exp(-\tfrac{1}{2}t^2)\,dt$$

$$when\ x \geq 2 \quad erfc(x) \cong \frac{1}{x\sqrt{2p}}(1 - \frac{1}{x^2})\exp(-\tfrac{1}{2}x^2)$$

$$C(x) = \int_0^x \sin\left(\frac{p}{2}t^2\right)dt$$

$$S(x) = \int_0^x \sin\left(\frac{p}{2}t^2\right)dt$$

$$C(x) = \to \tfrac{1}{2} \quad S(x) \to \tfrac{1}{2} \quad as\ x \to \infty$$

Tables of Integrals

Indefinite

$$\int \sin x \, dx = -\cos x$$

$$\int \frac{dx}{a^2 - x^2} = \sin^{-1} \frac{x}{a}$$

$$\int x \, e^{ax} dx = \frac{ax - 1}{a^2} e^{ax}$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$$

$$\int \sin^2 x \, dx = -1/2 \cos x \sin x + 1/2x = 1/2\, x - 1/4 \sin 2x$$

$$\int u \, dv = uv - \int v \, du \qquad \text{(integration by parts)}$$

Definite

$$\int_0^\infty \frac{x^{m-1}}{1 + x^n} dx = \frac{p / n}{\sin(mp / n)} \qquad n > m > 0$$

$$\int_0^\infty \frac{\sin x}{x} dx = \int_0^\infty \frac{\tan x}{x} dx = \frac{p}{2}$$

$$\int_0^\infty \left(\frac{\sin x}{x} \right)^2 dx = \frac{p}{2}$$

$$\int_0^\infty \frac{\cos nx}{1 + x^2} dx = \frac{p}{2} e^{-|n|}$$

$$\int_0^\infty x \, e^{-x^2} dx = \frac{1}{2}$$

$$\frac{1}{T_0} \int_{-t/2}^{t/2} A \cos\left(\frac{2pnt)}{T_0} \right) dt = \frac{At}{T_0} \operatorname{sinc}\left(\frac{nt}{T_0} \right) \qquad \text{where} \quad \operatorname{sinc}(u) = \frac{\sin p u}{p u}$$

Laplace and Z Transforms

$f(t)$	$F(s)=L[f(t)]$	$f_k = f(k\Delta)$	$F^d(z)=Z[f_k]$
1	$\frac{1}{s}$	1	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$k\Delta$	$\frac{\Delta z}{(z-1)^2}$
$\frac{t^n}{n!}$	$\frac{1}{s^{n+1}}$	$\frac{(k\Delta)^n}{n!}$	$\frac{\Delta^n}{n!}\left(-z\frac{d}{dz}\right)^n \frac{z}{z-1}$
e^{-at}	$\frac{1}{s+a}$	$e^{-a\Delta k}$	$\frac{z}{z-e^{-a\Delta}}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\Delta k e^{-a\Delta k}$	$\frac{\Delta e^{-a\Delta} z}{(z-e^{-a\Delta})^2}$
$\frac{t^{n-1}e^{-at}}{(n-1)!}$	$\frac{1}{(s+a)^n}$	$\frac{(\Delta k)^{n-1}e^{-a\Delta k}}{(n-1)!}$	$\frac{\Delta^{n-1}}{(n-1)!}\left(-z\frac{d}{dz}\right)^{n-1}\frac{z}{z-e^{-a\Delta}}$
$\cos w\ t$	$\frac{s}{s^2+w^2}$	$\cos w\ \Delta k$	$\frac{z(z-\cos w\ \Delta)}{z^2-2z\cos w\Delta+1}$
$\sin w\ t$	$\frac{w}{s^2+w^2}$	$\sin w\ \Delta k$	$\frac{z \sin w\ \Delta}{z^2-2z\cos w\Delta+1}$
$\cosh w\ t$	$\frac{s}{s^2-w^2}$	$\cosh w\ \Delta k$	$\frac{z(z-\cosh w\ \Delta)}{z^2-2z\cosh w\Delta+1}$
$\sinh w\ t$	$\frac{a}{s^2-w^2}$	$\sinh w\ \Delta k$	$\frac{z \sinh w\ \Delta}{z^2-2z\cosh w\Delta+1}$
$e^{-at}\sin w\ t$	$\frac{w}{(s+a)^2+w^2}$	$e^{-a\Delta k}\sin w\ \Delta k$	$\frac{z\ e^{-a\Delta}\sin w\ \Delta}{z^2-2ze^{-a\Delta}\cos w\Delta+e^{-2a\Delta}}$
$e^{-at}\cos w\ t$	$\frac{s+a}{(s+a)^2+w^2}$	$e^{-a\Delta k}\cos w\ \Delta k$	$\frac{z(z-e^{-a\Delta}\cos w\Delta)}{z^2-2ze^{-a\Delta}\cos w\Delta+e^{-2a\Delta}}$
$e^{-at}f(t)$	$F(s+a)$	$e^{-a\Delta k}f(k\Delta)$	$F^d(e^{a\Delta}z)$
$f(t-n\Delta)$	$e^{-sn\Delta}F(s)$	f_{k-n}	$z^{-n}F^d(z)$

Properties of Laplace and Z Transforms

Laplace Transform		Z Transform	
f(t)	***F(s)***	***f_k***	***$F^d(z)$***
1. Linearity			
$af(t)+bg(t)$	$aF(s)+bG(s)$	af_k+bg_k	$aF^d(z)+bG^d(z)$
2. Time Differentiation		**Differences**	
$f^{(n)}(t)=\frac{d^n f(t)}{dt^n}$	$s^nF(s)-\sum_{i=0}^{n-1}s^{n-1-i}f^{(i)}(0)$	**(a) Forward** $\Delta^n f_k$	$(z-1)^nF^d(z)-z\sum_{i=0}^{n-1}(z-1)^{n-1-i}\Delta^i f_0$
		(b) Backward $\nabla^n f_k$	$(1-z^{-1})^nF^d(z)$
3. Integration		**Sums**	
$f^{(-1)}(t)=\int_{-\infty}^{t}f(l)dl$	$\frac{F(s)}{s}+\frac{1}{s}f^{(-1)}(0)$	$\sum_{k=0}^{n}f_k$	$\frac{1}{1-z^{-1}}F^d(z)$
or $\int_{0}^{t}f(l)dl$	$\frac{F(s)}{s}$		
4. Translation			
$f(t-a)$, *where* $f(t)=0$ *when* $t<0$	$F(s)e^{-as}\quad a>0$	f_{k+i}	$z^iF^d(z)-z^i\sum_{j=0}^{i-1}z^{-j}f_j$
5. Multiplication by e^{-at}		**Multiplication by a^k**	
$f(t)e^{-at}$	$F(s+a)$	$a^k f_k$	$F^d(a^{-1}z)$

Properties of Laplace and Z Transforms (Continued)

6. Multiplication by t^n

$t^n f(t)$

$(-1)^n \dfrac{d^n F(s)}{ds^n}$

Multiplation by k^n

$k^n f_k$

$\left(-z \dfrac{d}{dz}\right)^n F^d(z)$

7. Convolution

$\int_0^t f(t-l)g(l)dl$

$F(s)\,G(s)$

$\sum_{k=0}^{n} f_k \; g_{n-k}$

$F^d(z)\,G^d(z)$

8. Product of two functions

$f(t)\;g(t)$

$\dfrac{1}{2pj}\int_{c-j\infty}^{c+j\infty} F(s-w)G(w)dw,\; j=\sqrt{-1}$

$f_k \; g_k$

$\dfrac{1}{2pj}\oint_c w^{-1}F(w)G(w^{-1}z)dw$

9. Initial Value

$f(0)=\lim_{s\to\infty} sF(s)$

$f(0)=\lim_{z\to\infty} F(z)$

10. Final Value

$\lim_{t\to 0} f(t)=\lim_{s\to 0} sF(s)$

$\lim_{k\to\infty} f_k = \lim_{z\to 1}(1-z^{-1})\,F(z)$

Fourier Series and Transforms

Fourier Series - Trigonometric Form

If $f(t)$ is periodic with period T then it can be expressed as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\{a_n \cos nw_0 t + b_n \sin nw_0 t\},$$

$$w_0 = \frac{2p}{T}$$

where

$$a_0 = \frac{2}{T}\int_0^T f(t)\, dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos nw_0 t\, dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin nw_0 t\, dt$$

Fourier Series - Complex Form

If $f(t)$ is periodic with period T then

$$f(t) = \sum_{n=-\infty}^{\infty} F_n\, e^{jnw_0 t}$$

where

$$F_n = \frac{1}{T}\int_0^T f(t)\, e^{-jnw_0 t}$$

Fourier Transform

If $f(t)$ is aperiodic then

$$f(t) = \frac{1}{2p}\int_{-\infty}^{\infty} F(w) e^{jwt} dw = F^{-1}[F(w)]$$

where

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt}\, dt = F[f(t)]$$

Properties of the Fourier Transform

$$F\left[\frac{d}{dt} f(t)\right] = jwF(w)$$

$$F\left[\frac{d^n}{dt^n} f(t)\right] = (jw)^n F(w)$$

$$F\left[f(t-t_0)\right] = \exp(-jw\, t_0)F(w)$$

$$F\left[\int_{-\infty}^{t} f(t)dt\right] = \frac{F(w)}{jw} + pF(0).d(w)$$

where $d(w)$ *is the Dirac impulse function*

$$\int_{-\infty}^{\infty} f(t)h(t-t)dt = F(w)H(w)$$

Common Fourier Transforms

$f(t)$	$F(w)$	
K	$2p\, K\, d(w)$	
$K\, d(t)$	K	
$A\left[u(t+\frac{T}{2}) - u(t-\frac{T}{2})\right]$	$A\, T\, \frac{\sin wT/2}{wT/2}$	*where* $u(t)$ *is the Heaviside step function*
$\sin w_0 t$	$jp\, \left[d(w+w_0) - d(w-w_0)\right]$	
$\cos w_0 t$	$p\, \left[d(w+w_0) - (w-w_0)\right]$	
$e^{-\lvert a \rvert} t$	$\frac{2a}{a^2+w^2}$	
$\left(\frac{a}{p}\right)^{1/2} e^{-at^2}$	$2p\, e^{-w^2/4a}$	

Vector Analysis

The rectangular *(x,y,z)*, cylindrical *(r, f, z)* and spherical coordinates *(r, q, f)* are related as follows:

$$
\begin{aligned}
x &= r\ \cos f = r\sin q\ \cos f \\
y &= r\ \sin f = r\sin q\ \ \sin f \\
z &= r\ \cos q \\
r &= \sqrt{x^2 + y^2} = r\sin q \\
f &= \mathrm{atan}(y/x) \\
r &= \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2} \\
q &= \mathrm{atan}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) = \mathrm{atan}(\frac{r}{z})
\end{aligned}
$$

The components of a vector *F* is one coordinate systems transforms to the other coordinate system as follows

$$
\begin{aligned}
F_x &= Fr\ \cos f - F_f \sin f \\
&= F_r \sin q\ \cos f + F_q \cos q\ \cos f - F_f\ \sin f \\
F_y &= F_r\ \sin f + F_f\ \cos f \\
&= F_r \sin q\ \sin f + F_q \cos q\ \sin f + F_f \cos f \\
F_z &= F_r \cos q - F_q \sin q \\
F_r &= F_x \cos f + F_y \sin f \\
&= F_r \sin q + F_q \cos q \\
F_f &= -F_x \sin f + F_y \cos f \\
F_r &= F_x \sin q \cos f + F_y \sin q\ \sin f + F_z \cos q \\
&= F_r\ \sin q + F_z \cos q \\
F_q &= F_x \cos q\ \cos f + F_y \cos q\ \sin f - F_z \sin q \\
&= F_r\ \cos q - F_z \sin q
\end{aligned}
$$

In Rectangular Coordinates

$$\nabla V = \underset{\sim}{\hat{x}}\frac{\partial V}{\partial x} + \underset{\sim}{\hat{y}}\frac{\partial V}{\partial y} + \underset{\sim}{\hat{z}}\frac{\partial V}{\partial z}$$

$$\nabla . \underset{\sim}{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla x \underset{\sim}{F} = \begin{vmatrix} \underset{\sim}{\hat{x}} & \underset{\sim}{\hat{y}} & \underset{\sim}{\hat{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

In Cylindrical Coordinates

$$\nabla V = \underset{\sim}{\hat{r}}\frac{\partial V}{\partial r} + \underset{\sim}{\hat{f}}\frac{1}{r}\frac{\partial V}{\partial f} + \underset{\sim}{\hat{z}}\frac{\partial V}{\partial z}$$

$$\nabla . \underset{\sim}{F} = \frac{1}{r}\frac{\partial}{\partial r}(r \; F_r) + \frac{1}{r}\frac{\partial F_f}{\partial f} + \frac{\partial F_z}{\partial z}$$

$$\nabla x \underset{\sim}{F} = \frac{1}{r}\begin{vmatrix} \underset{\sim}{\hat{r}} & r \underset{\sim}{\hat{f}} & \underset{\sim}{\hat{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial f} & \frac{\partial}{\partial z} \\ F_r & r \; F_f & F_z \end{vmatrix}$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r \; \frac{\partial V}{\partial r} \right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial f^2} + \frac{\partial^2 V}{\partial z^2}$$

In Sperical Cordinates

$$\nabla V = \underset{\sim}{\hat{r}}\frac{\partial V}{\partial r} + \hat{q}\frac{\partial V}{r\partial q} + \underset{\sim}{\hat{f}}\,\frac{1}{r\sin q}\frac{\partial V}{\partial f}$$

$$\nabla . \underset{\sim}{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin q}\frac{\partial}{\partial q}(\sin q\ F_q) + \frac{1}{r\sin q}\frac{\partial F_f}{\partial f}$$

$$\nabla x\, \underset{\sim}{F} = \frac{1}{r^2\sin q}\begin{vmatrix} \underset{\sim}{\hat{r}} & r\underset{\sim}{\hat{q}} & r\sin q\ \underset{\sim}{\hat{f}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial q} & \frac{\partial}{\partial f} \\ F_r & rF_q & r\sin q\ F_f \end{vmatrix}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin q}\frac{\partial}{\partial q}\left(\sin q\ \frac{\partial V}{\partial q}\right) + \frac{1}{r^2\sin^2 q}\frac{\partial^2 V}{\partial f^2}$$

Vector Identities

1. $\underset{\sim}{A}\times\left(\underset{\sim}{B}\times\underset{\sim}{C}\right)=(\underset{\sim}{A}.\underset{\sim}{C})\,\underset{\sim}{B}-(\underset{\sim}{A}.\underset{\sim}{B})\,\underset{\sim}{C}$

2. $\nabla.\nabla\times\underset{\sim}{F}=0$

3. $\nabla\times\nabla V=0$

4. $\nabla.(V\,\underset{\sim}{F})=V\;\nabla.\underset{\sim}{F}+\underset{\sim}{F}.\nabla V$

5. $\nabla\times(V\,\underset{\sim}{F})=V\;\nabla\times\underset{\sim}{F}+\nabla V\times\underset{\sim}{F}$

6. $\nabla.(\underset{\sim}{A}\times\underset{\sim}{B})=\underset{\sim}{B}.\nabla\times\underset{\sim}{A}-\underset{\sim}{A}.\nabla\times\underset{\sim}{B}$

7. $\nabla\times\nabla\times\underset{\sim}{F}=\nabla(\nabla.\underset{\sim}{F})-\nabla^2\,\underset{\sim}{F}$

8. $\nabla(\underset{\sim}{A}.\underset{\sim}{B})=(\underset{\sim}{A}.\nabla)\,\underset{\sim}{B}+(B.\nabla)\,\underset{\sim}{A}+\underset{\sim}{A}\times(\nabla\times\underset{\sim}{B})+B\times(\nabla\times\underset{\sim}{A})$

9. $\nabla\times(\underset{\sim}{A}\times\underset{\sim}{B})=\underset{\sim}{B}.\nabla\,\underset{\sim}{A}-\underset{\sim}{A}.\nabla B+\underset{\sim}{A}(\nabla.\underset{\sim}{B})-\underset{\sim}{B}(\nabla.\underset{\sim}{A})$