Analogue Modulation Techniques

Introduction

In analogue modulation techniques, the amplitude, frequency or Phase of a high frequency sinusoidal **carrier wave,** is varied in direct proportion to the **audio** or **message** signal. For Amplitude modulation, the amplitude of the carrier is varied. For Frequency Modulation, the frequency of the carrier is varied and for Phase modulation, the Phase of the carrier is varied. We will look at these techniques and their variations in this section.

AM modulation

Amplitude Modulation or AM is a simple form of modulation where the amplitude of the carrier is varied in proportion to the message signal. As a result if $X_m(t)$ is the audio message or modulating signal, A_c is the carrier amplitude, ω_c is the carrier frequency and m is a constant called the **modulation index**, then the Radio Frequency (RF) or modulated AM signal $X_c(t)$ can be written as:

$$X_c(t) = [1 + m.X_m(t)] . A_c . \cos \omega_c t \qquad \text{Eqn. 1}$$

For AM the modulation index (m) is always chosen such that;

$$-1 \leq m.X_m(t) \leq 1 \qquad \text{Eqn. 2}$$

So that $[1 + m.X_m(t)]$ will always be positive. Normally the input voltage $X_m(t)$ is normalised to have a maximum amplitude of 1. Under those conditions m is expressed as a percentage of the maximum allowable amplitude. If the modulation index is thus 30% then the maximum amplitude of $\{m.X_m(t)\}$ will be 0.3.

The block diagram corresponding to equation 1 can be realised using Visual System Simulator (VSS) as shown in the left hand side of figure 1. The DC or 1 in the $[1 + m.X_m(t)]$ term is obtained by using a pulsed waveform with a positive and negative pulse amplitude of 1, so that there is a DC offset and zero pulse amplitude. The $m.X_m(t)$ term is obtained by having one or more sinewave sources. In this example a sinewave and its second harmonic, with independent control over the fundamental and second harmonic amplitude is used for modulating signal. The frequency of the fundamental sinewave is 1 kHz. These three signals are added and then multiplied with a carrier of 32 kHz. The resulting waveforms are shown in figure 2.

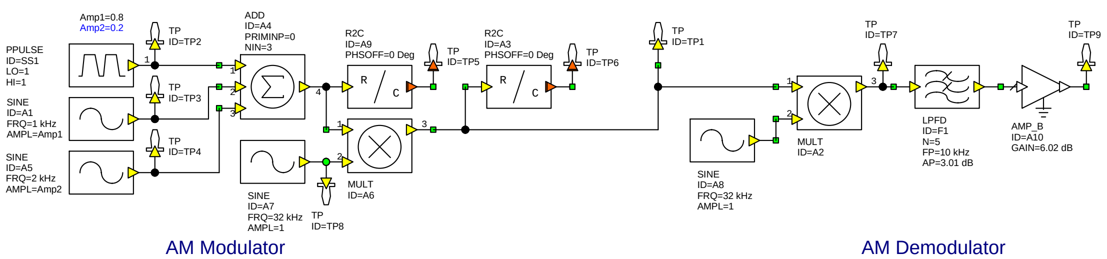


Figure 1. Block Diagram of AM Modulator and demodulator

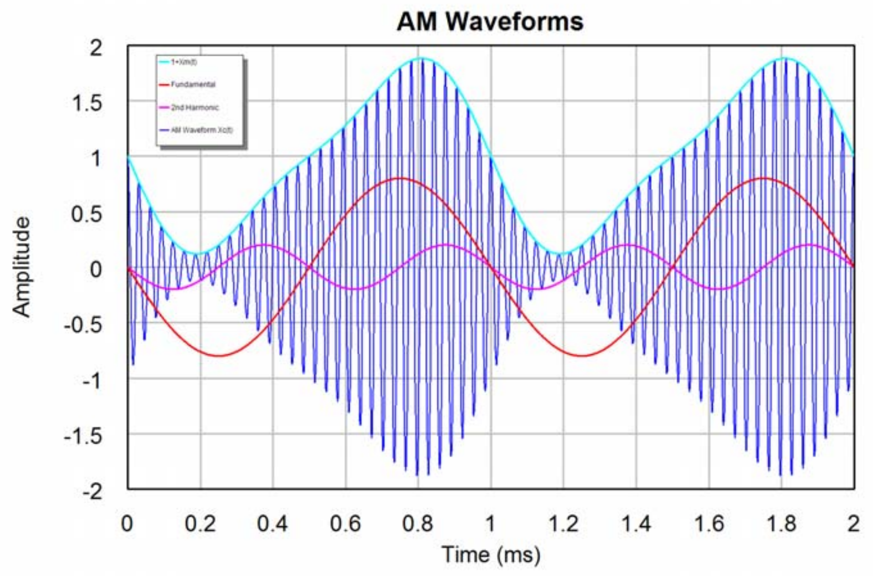


Figure 2a. Audio and RF Waveforms for Amplitude Modulation.

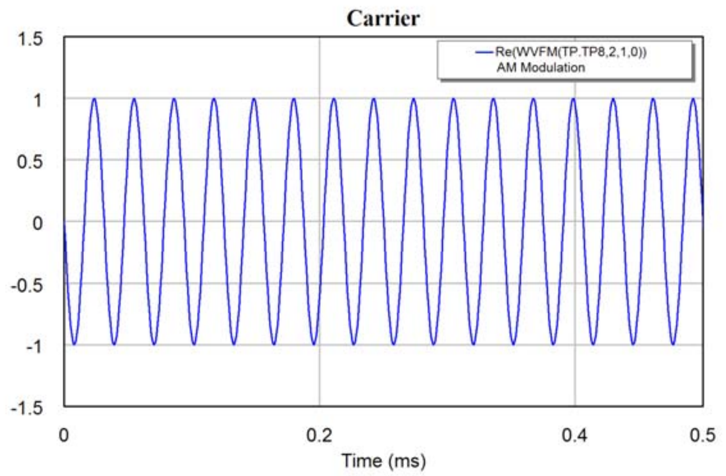


Figure 2b. Carrier waveform

Note the message signal is the "envelope" of the AM signal.

AM Spectrum

Consider the case when the message is a single tone with constant amplitude.

So that: $X_m(t) = A_m \cos \omega_m t$ Eqn. 3

Then the resulting AM signal will be given by:

$$X_c(t) = [1 + m.\ A_m \cos \omega_m t]\ .\ A_c\ .\ \cos \omega_c t \qquad \text{Eqn. 4}$$

$$= A_c\ .\ \cos \omega_c t + \frac{m\, A_m A_c}{2} \cos [(\omega_c + \omega_m)\, t]$$

$$+ \frac{m\, A_m A_c}{2} \cos [(\omega_c - \omega_m)\, t] \qquad \text{Eqn. 5}$$

The spectrum is then found by taking the Fourier transform of $X_c(t)$.

Remembering that the Fourier Transform (A cos ω_ot) = $\frac{A}{2}[\delta(\omega - \omega_o) + \delta(\omega + \omega_o)]$ where δ is the Dirac Delta function. In terms of a spectrum then, a cosine signal with a frequency of ω_o radians/sec, is represented as two lines at frequencies of $\pm\omega_o$.

The frequency components of the AM signal shown in equation 5, consists of three cosine signals, resulting in three lines in the positive frequency domain and three similar level spikes in negative frequency as shown in figure 3.

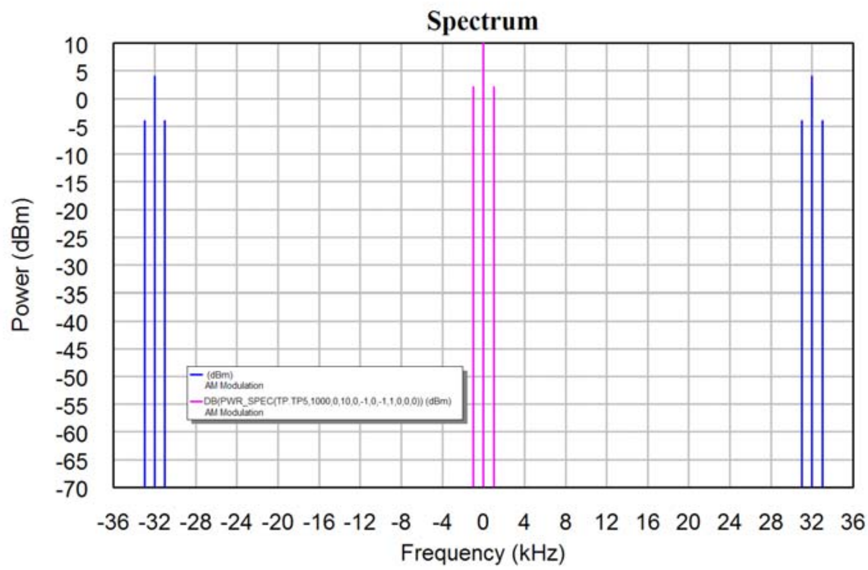


Figure 3 : AM spectrum for single sinewave modulation.

The magenta spectral lines in figure 3 show the modulating (audio) spectrum, including the DC component that is added to ensure that $[1 + m.X_m(t)]$ is always > 0. The magenta spectrum will thus correspond to $[1 + m.X_m(t)]$. The blue lines show the modulated (AM) spectrum and correspond to $[1 + m.X_m(t)]Sin(\omega_c t)$.

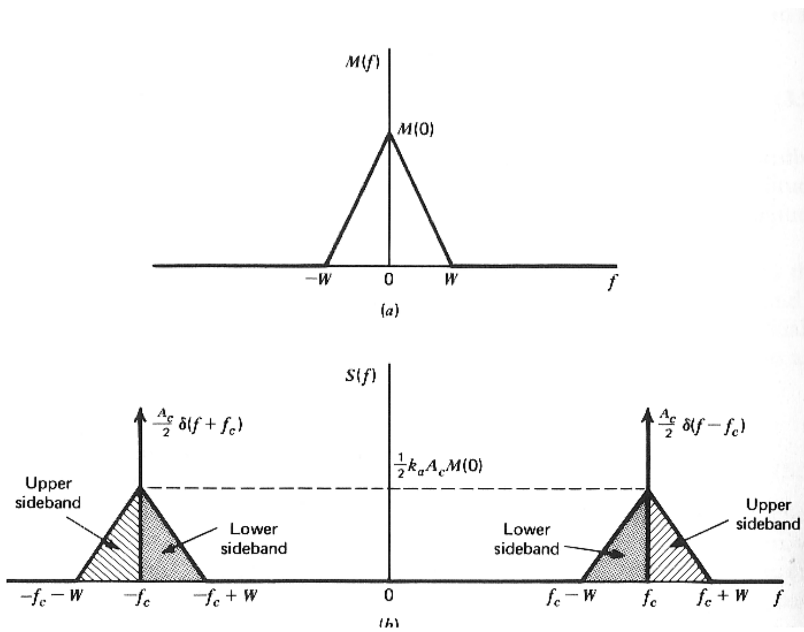


Figure 4 (a) Message (b) Resulting AM spectrum.

Normally the modulating signal is much more complex than this. Assume then that out message had a spectrum as shown in Figure 4(a). Using the principle of superposition, the resulting AM spectrum would be like in Figure 4(b).

Notes about AM

- The bandwidth occupied by the AM signal is twice that used by the message. The AM signal consists of identical size **upper and lower sidebands.**
- The message spectrum is shifted to around the carrier frequency, fc. As usual, the spectrum is repeated in negative frequency.
- The AM spectrum always contains a large amplitude spike at the carrier frequency, fc.
- The carrier in the AM spectrum corresponds to the dc component of the message and therefore contains no real information. This component represents wasted power.

Amplitude modulation is also called “Linear” modulation as the RF spectrum is a linear translation

Power in AM Carrier and Sidebands

Assume the message signal is a single tone like before. The AM signal can be expressed mathematical as in equation 5.

$$X_c(t) = A_c \cdot \cos \omega_c t + \frac{m\,A_m A_c}{2} \cos[(\omega_c + \omega_m)\,t] + \frac{m\,A_m A_c}{2} \cos[(\omega_c - \omega_m)\,t] \qquad \text{Eqn. 6}$$

Voltage	$\frac{A_c}{\sqrt{2}}$	$\frac{mA_cA_m}{2\sqrt{2}}$	$\frac{mA_cA_m}{2\sqrt{2}}$
Power	$\frac{A_c^2}{2}$	$\frac{(mA_cA_m)^2}{8}$	$\frac{(mA_cA_m)^2}{8}$

Thus the total power in the carrier is $\frac{A_c^2}{2}$ while the total power contained in the two sidebands is $\frac{(mA_cA_m)^2}{4}$.

Since $|mA_m| \le 1$, the carrier power is always greater than the sideband power. The maximum sideband power occurs when $|mA_m|=1$ but even then the total sideband power will be ½ of the carrier power! **AM is inefficient** in terms of power usage, but as can be seen later, it allows low cost receivers to be used. Most of the power (that from the carrier) is simply used to announce that there is an AM radio station at that carrier frequency.

In general, the efficiency of AM is given by the total sideband power to the total power transmitted and will be given by;

$$\eta = \frac{\text{Sideband Power}}{\text{Carrier Power} + \text{Sideband Power}} = \frac{m^2}{2+m^2} \qquad \text{Eqn. 7}$$

where m in this expression is the normalised modulation index.

Generation and Demodulation of AM

Different techniques will be discussed during the lectures, these techniques are described in standard texts. They include:

1. High power modulation to obtain good efficiency
2. Low power modulation to obtain low distortion
3. Envelope detection
4. Synchronous detection

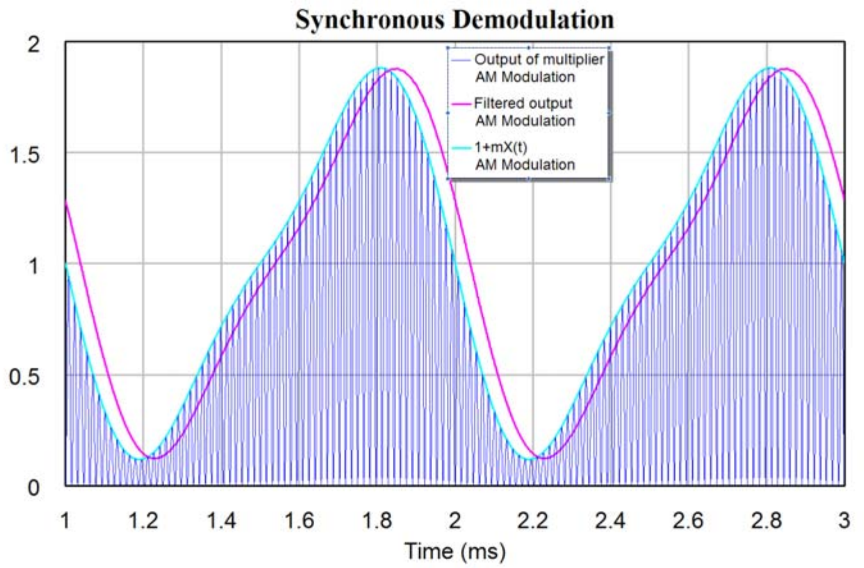


Figure 5. Synchronous demodulation waveforms for modulating signal of figure 1.

Other Linear Modulation Schemes

Double Sideband Suppressed Carrier Modulation (DSB-SC or DSB)

For DSB, the 1 term in $[1 + m.X_m(t)]$ is removed so that there is no large "carrier" component in the spectrum. To produce DSB, the message signal is just multiplied by the carrier signal. The block diagram of figure 1, is thus changed to that of figure 6.

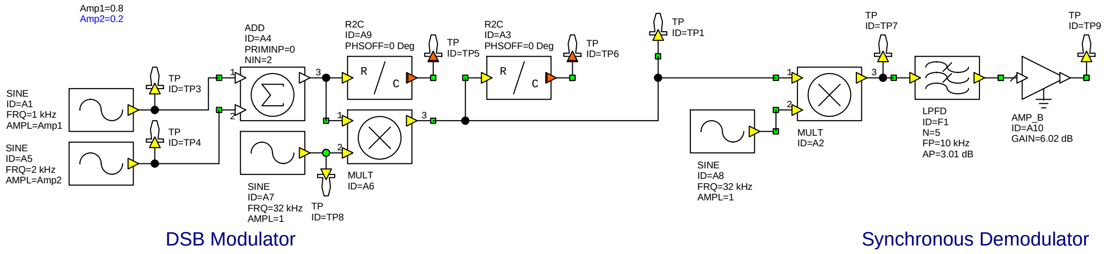


Figure 6. DSB generation and demodulation

If the modulating signal is m $X_m(t)$ and the carrier by $A_c \cos(\omega_c t)$, then the resulting DSB signal is:

$$X_c(t) = m X_m(t) A_c \cos(\omega_c t)$$

If $X_m(t)$ is a single tone, for example $A_m \cos(\omega_m t)$, then the DSB signal will be;

$$X_c(t) = mA_m \cos(\omega_m t) A_c \cos(\omega_c t)$$
$$= \frac{mA_m A_c}{2}[\cos\{(\omega_c - \omega_m)t\} + \cos\{(\omega_c + \omega_m)t\}] \qquad \text{Eqn. 8}$$

The resulting waveforms are shown in figure 7.

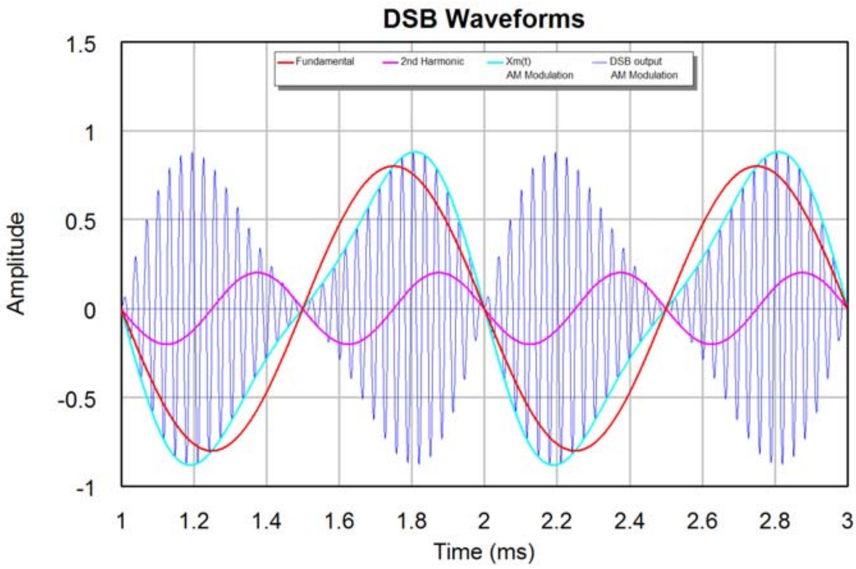


Figure 7. DSB waveforms for the modulating signal of figure 2.

The spectrum is shown in figure 8 and corresponds to the AM spectrum shown in figure 3. Notice that there is no carrier frequency component in the modulated signal or DC component in the modulating signal.

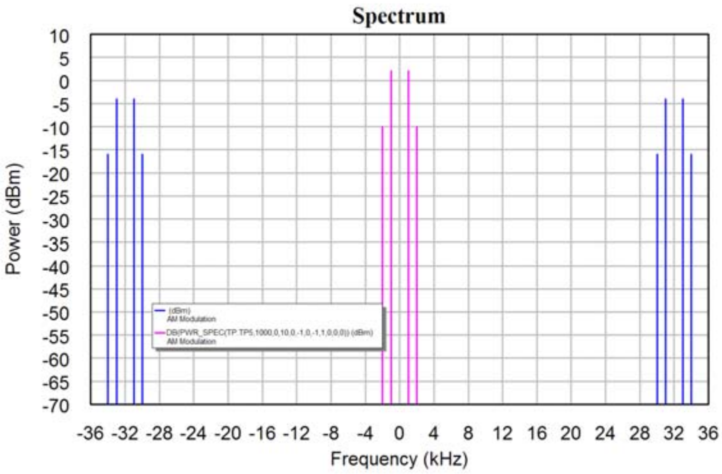


Figure 8. DSB spectrum.

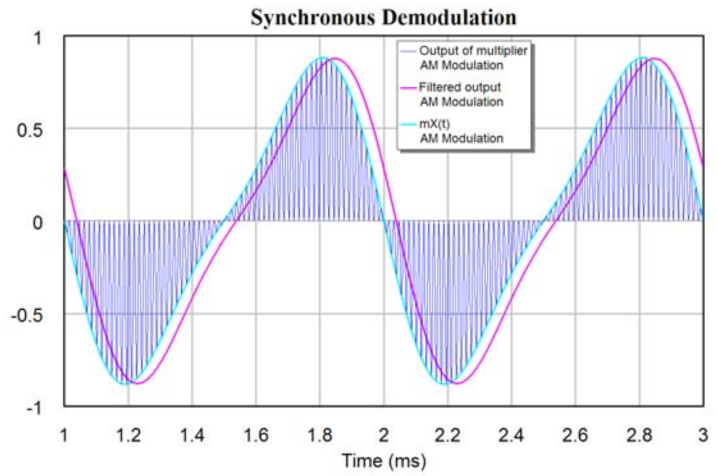


Figure 9. DSB demodulation

Figure 9 shows the waveforms for the synchronous demodulation of figure 6. This is for the same input waveform shown in figure 5. Note the difference in the DC component at the demodulator.

Advantages

- Don't waste power sending a carrier signal which contains no message information

Disadvantage

- Must use a synchronous detector

Single Sideband Modulation (SSB)

The easiest way to imagine how we could produce Single Sideband Modulation is to start with DSB and then remove one sideband. Note both sidebands carry the same message information and thus one sideband is redundant. The advantage of removing one sideband is that the modulate signal uses ½ as much spectrum as AM or DSB. SSB is normally only used for speech applications since the phase shifts produced during the modulation and demodulation process change the waveforms significantly. The ears are insensitive to phase.

Advantage

- Uses ½ the bandwidth
- Can demodulate by multiplying with a local oscillator. That does not need to be synchronised with the transmitted carrier in either frequency or phase. Having a slight frequency shift changes all demodulated frequencies with that same frequency shift.

Techniques:

1 Filter to remove one sideband. This is not practical since filters with extreme transition bands are required.

2 Hilbert transform.

3 Successive mixing and filtering (also known as the "third method")

Hilbert Transform

The definition of the Hilbert transform function is:

$$H(f) = -jSign(f) \quad \text{Eqn. 9}$$

Fro positive frequencies H(f) = -j and for negative frequencies H(f) = j. H(f) corresponds to an amplitude independent phase shift of -90 degrees. Using the Laplace transform, the impulse response can be determined as:

$$H(t) = \frac{1}{\pi t} \quad \text{Eqn. 10}$$

This function knows that an impulse will be coming in a little while and already takes action. Clearly this is not a realisable function, but it can be approximated using FIR digital filtering techniques and incorporating a delay.

The positive frequency spectrum is:

$X_+(f) = X(f) \qquad f > 0$ Eqn. 11

$X_+(f) = 0 \qquad f < 0$

So that:

$X_+(f) = ½(1+sign(f))\ X(f)$

$$X_+(f) = \frac{1}{2}X(f) + \frac{1}{2}jX(f)H(f) = \frac{1}{2}X(f) + \frac{1}{2}j\hat{X}(f) \quad \text{Eqn. 12}$$

Where $\hat{X}(f)$ is the Hilbert Transform of the function $H(f)$. Similarly:

$$X_+(t) = \frac{1}{2}X(t) + \frac{1}{2}j\hat{X}(t) \quad \text{Eqn. 13}$$

Similarly a negative only frequency spectrum is:

$$X_-(t) = \frac{1}{2}X(t) - \frac{1}{2}j\hat{X}(t) \quad \text{Eqn. 13}$$

Equation 13 is a complex function of time. So to have a +ve or –ve only frequency spectrum, complex time functions are required. In many DSP applications these complex waveforms can be generated digitally.

An Upper Sideband signal consists of:

$$X_{USB}(t) = \frac{1}{2}[e^{(-j\omega_c t)}X_-(t) + e^{(j\omega_c t)}X_+(t)] \quad \text{Eqn. 14}$$

$$X_{USB}(t) = \frac{1}{2}[e^{(-j\omega_c t)}\{\frac{X(t) - j\hat{X}(t)}{2}\} + e^{(j\omega_c t)}\{\frac{X(t) + j\hat{X}(t)}{2}\}] \quad \text{Eqn. 15}$$

$$X_{USB}(t) = \frac{1}{2}[X(t)\{\frac{e^{(-j\omega_c t)} + e^{(j\omega_c t)}}{2}\} - \hat{X}(t)\{\frac{e^{(-j\omega_c t)} + e^{(j\omega_c t)}}{2j}\}] \quad \text{Eqn. 16}$$

$$X_{USB}(t) = \frac{1}{2}[X(t)\cos(\omega_c t) - \hat{X}(t)\sin(\omega_c t)] \quad \text{Eqn. 17}$$

Eqn 17 is a real time function. Similarly the lower sideband can be obtained as:

$$X_{LSB}(t) = \frac{1}{2}[X(t)\cos(\omega_c t) + \hat{X}(t)\sin(\omega_c t)] \qquad \text{Eqn. 17}$$

In figure 10 the Hilbert transform function is realised using 1 kHz sinewaves for the modulating signal. The top modulator is the I (In Phase) modulator, using the cosine carrier waveform. The bottom modulator is the Q (Quadrature) modulator, using a sine carrier waveform. The I and Q modulators have a fundamental (1 kHz) and second harmonic (2 kHz) component. For the I modulator these are both Cosine waveforms and for the Q modulator they are both Sine waveforms. The Q modulating (audio) waveforms are thus the Hilbert transform of the I modulating waveforms.

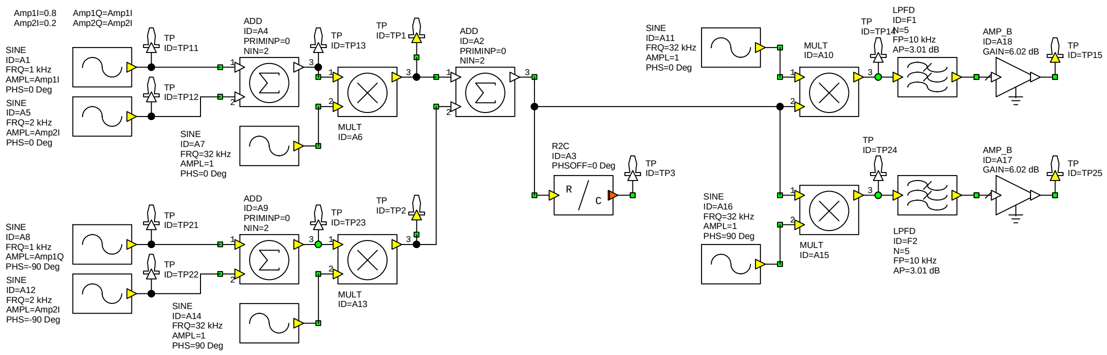


Figure 10. Blocks diagram for an SSB modulator using the Hilbert Transform.

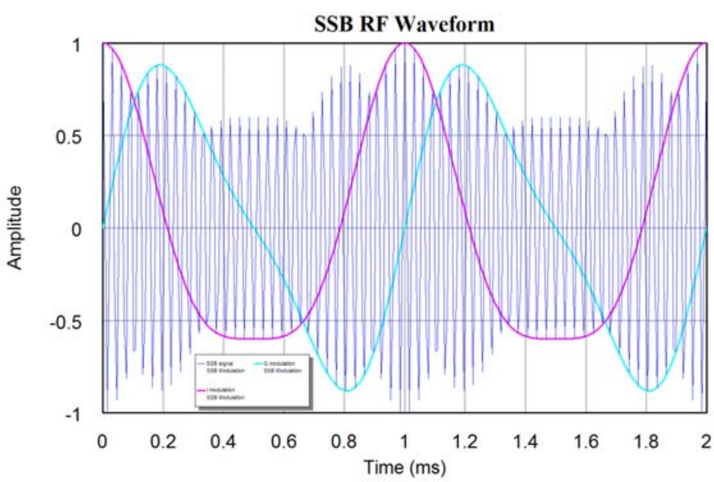


Figure 11. RF waveform for SSB signal from block diagram of figure 10.

The SSB spectrum shown in figure 12 should be compared with the same spectrum for AM shown in figure 3 and that for DSB shown in figure 8. In figure 12, the magenta lines are the baseband spectrum. Note that in this case no DC component, which will result in a carrier at the RF spectrum is implemented. The blue lines are the corresponding RF spectrum of the SSB signal. Note that no lower sideband signal is present.

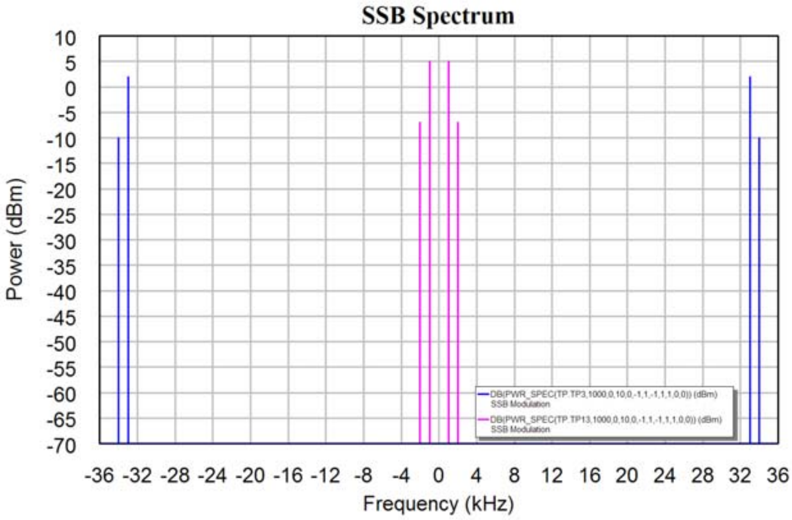


Figure 12. SSB spectrum from block diagram of figure 10.

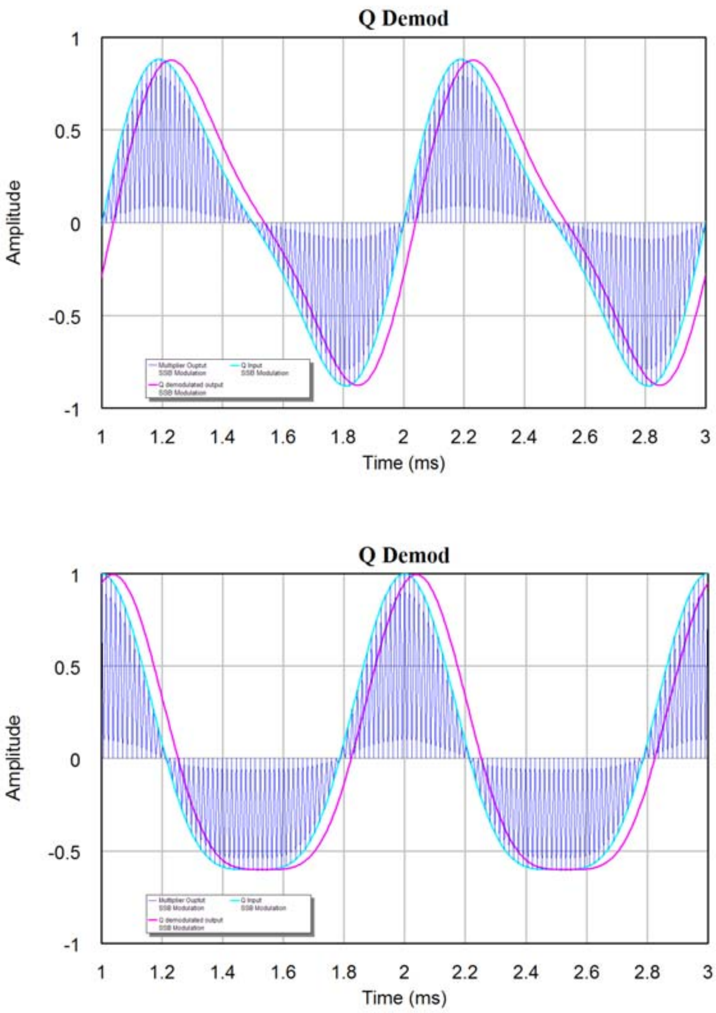


Figure 13. I and Q demodulated output signals.

Both the I and Q demodulated output signals have the same spectrum. There is a phase difference in the second harmonic, which results in the waveform looking different. To our ears these waveforms sound the same.

A Carrier can be used in an SSB system. That will then allow envelope detection to be used for the demodulation and overcomes the problem of requiring a local oscillator which is the same as the transmitted oscillator.

Vestigial Side Band (VSB) Modulation

One of the problems with SSB is that a very "sharp" cut-off filter is required to remove all of one sideband and the carrier without effecting the message carrying sideband. In practice, some of the low frequency components of the wanted sideband may be effected. SSB is not good when we want to send dc or low frequency message components. In Vestigial Side Band (VSB) modulation the strict requirements on the SSB filter are relaxed so a more practical filter can be implemented. In VSB, the spectrum is similar to SSB but a vestige of the "unwanted" sideband is retained.

A typical VSB spectrum showing both positive and negative frequency components is shown in Figure 7.

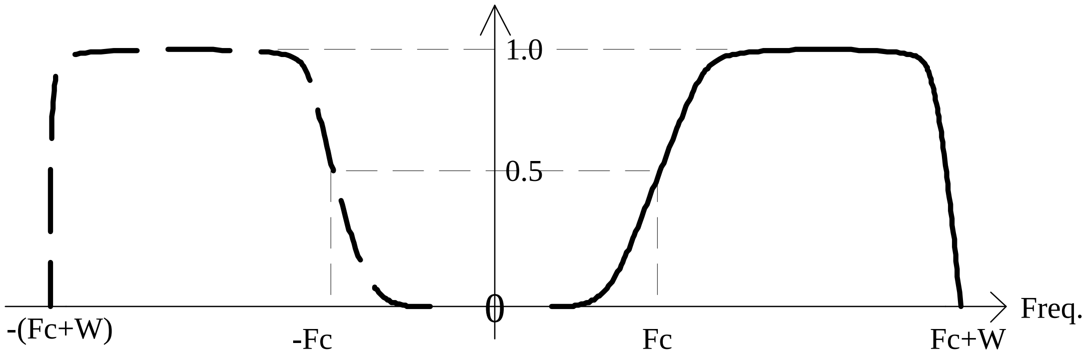


Figure 14.

Note that the filter response is designed such that exactly ½ of the carrier component (Fc) is retained. The response is also almost linear along this edge. If the VSB signal is multiplied by the carrier frequency in a mixer we have the negative frequency component moving up Fc hertz while the positive frequency component slides down Fc hertz. The combined response is like shown in Figure 8.

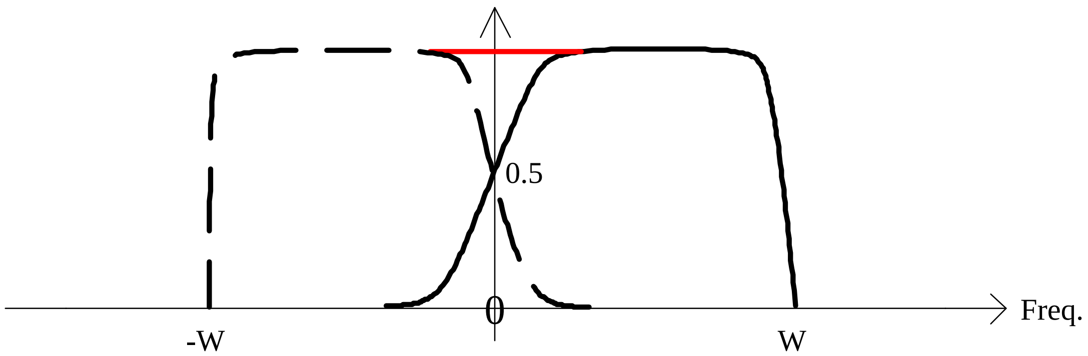


Figure 15.

Note the overlapping parts of the positive and negative frequency components sum to give a total response of 1 (shown in red). The recovered message is therefore not distorted.

Advantages

- VSB filter does not need to have very fast roll-off like in SSB filtering.
- Can send dc and low frequencies efficiently.
- If a carrier component is added to VSB, it can be demodulated with an envelope detector.

Disadvantages

- Slight increase in bandwidth compared with SSB.

VSB with a carrier is used in analogue TV systems.

Quadrature Modulation

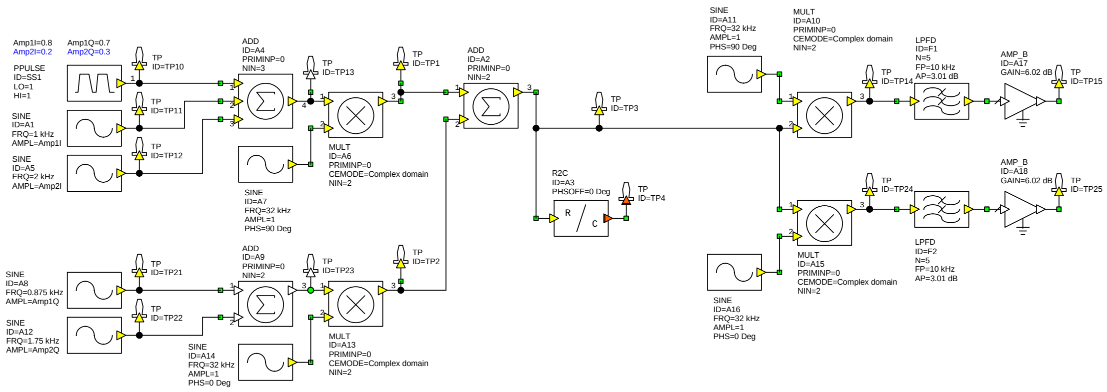


Figure 16. Block diagram of an IQ modulation and demodulation system.

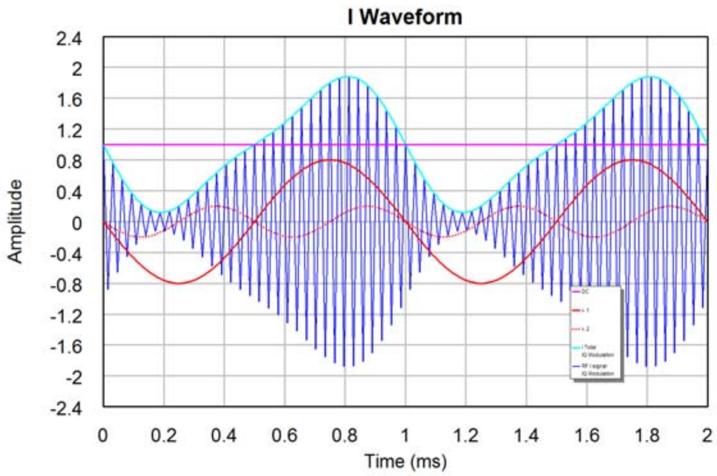


Figure 17a. I waveform for system in figure 16.

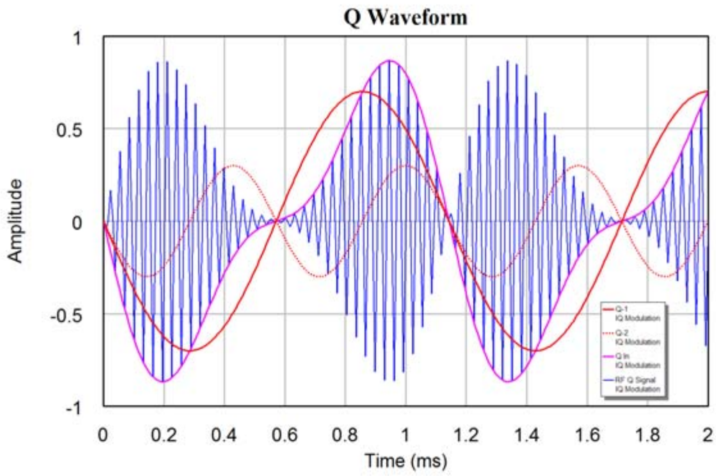


Figure 17b. Q waveform for system in figure 16.

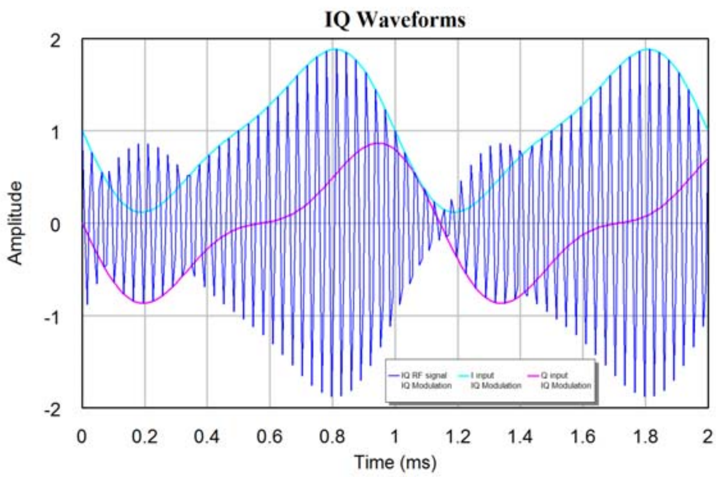


Figure 17c. I-Q RF waveform for system in figure 16.

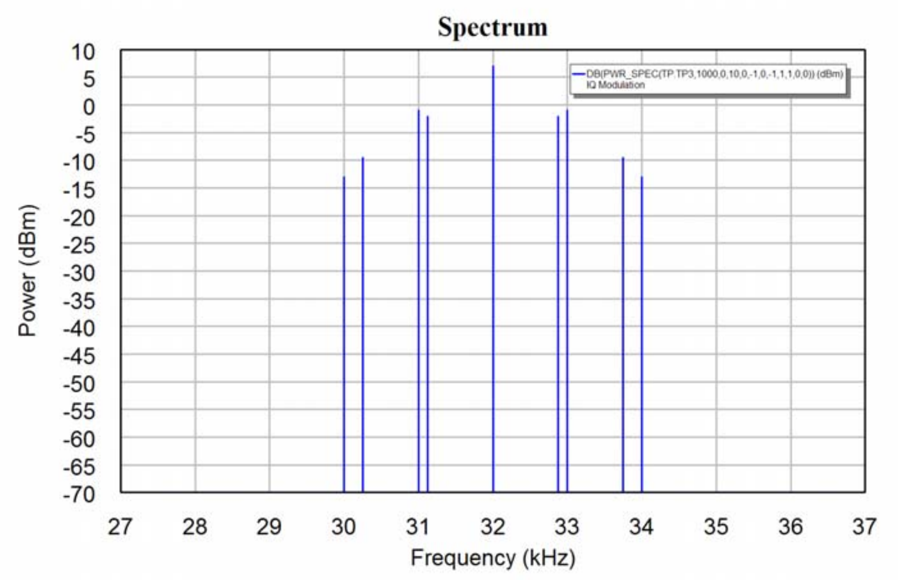


Figure 18. Spectrum of IQ waveform of figure 17c.

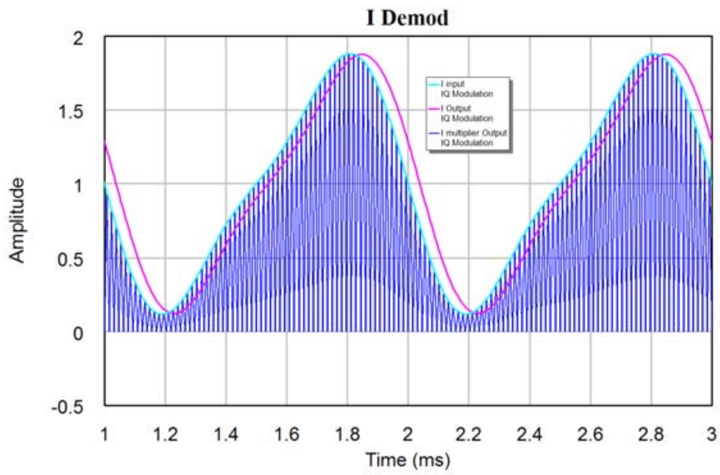


Figure 19a. Demodulated I waveform for system in figure 16.

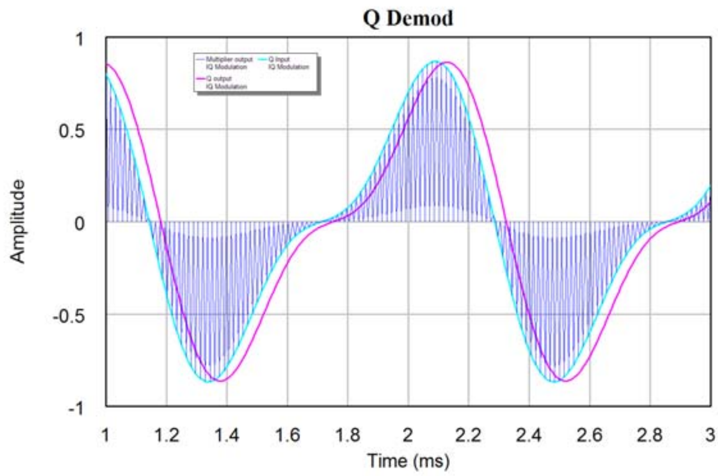


Figure 19b. Demodulated Q waveform for system in figure 16.

Exponential Modulation (FM and PM)

In these modulation schemes the phase or frequency of the carrier is varied by the audio. The carrier amplitude remains the same. Note, because frequency is just the rate of change of phase, we would expect that PM and FM are quite similar. As we will see, this is in fact the case.

Phase Modulation

Let the audio signal be Xm(t), so the modulated signal becomes

$$X_{PM}(t) = Ac \cos[\omega_c t + \phi_d \,.\, Xm(t)] \qquad \text{Eqn. 18}$$

where ϕ_d is a constant relating volts to phase (units radians per volt say). In this case, we usually choose $|Xm(t)| \leq 1$ and $\phi_d \leq \pi$, so that the maximum phase variation is always less than $\pm\pi$ so no phase ambiguity results.

Frequency Modulation

Consider a general carrier wave

$$X_c(t) = Ac \cos[\omega_c t + \phi(t)] \qquad \text{Eqn. 19}$$

where $\phi(t)$ is related to the audio modulating signal BUT what is the relationship?

The instantaneous frequency is the rate of change of the cosine phase. That is,

$$\text{Instantaneous Frequency} = \frac{\partial}{\partial t}[\omega_c t + \phi(t)]$$

$$= \omega_c + \frac{\partial}{\partial t}[\phi(t)] \qquad \text{Eqn. 20}$$

In FM we wish the instantaneous frequency to be linearly related to the audio voltage. That is,

$$\text{Instantaneous Frequency} = \omega_c + k_\omega \,.\, Xm(t)$$

where k_ω is a constant relating volts to frequency in radians/second [units of (radian/second)/volt say].

These two expressions (11) and (12) must be equal so

$$\frac{\partial}{\partial t}[\phi(t)] = k_\omega \,.\, Xm(t) \qquad \text{Eqn. 21}$$

or $$\phi(t) = \int_0^t k_\omega \,.\, Xm(\gamma)d\gamma \quad \text{where } \gamma \text{ is a dummy variable.} \qquad \text{Eqn. 22}$$

Thus the mathematically form of an FM signal is

$$X_{FM}(t) = A_c \cos(\omega_c t + k_\omega \int_0^t X_m(\gamma)\, d\gamma) \qquad \text{Eqn. 23}$$

Comparing equations 18 and 23 we note that PM and FM are closely related. In fact, an FM modulator can be built using a phase modulator by simply integrating the audio signal before applying it to the phase modulator, as shown in figure 20.

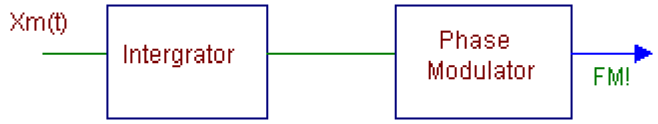
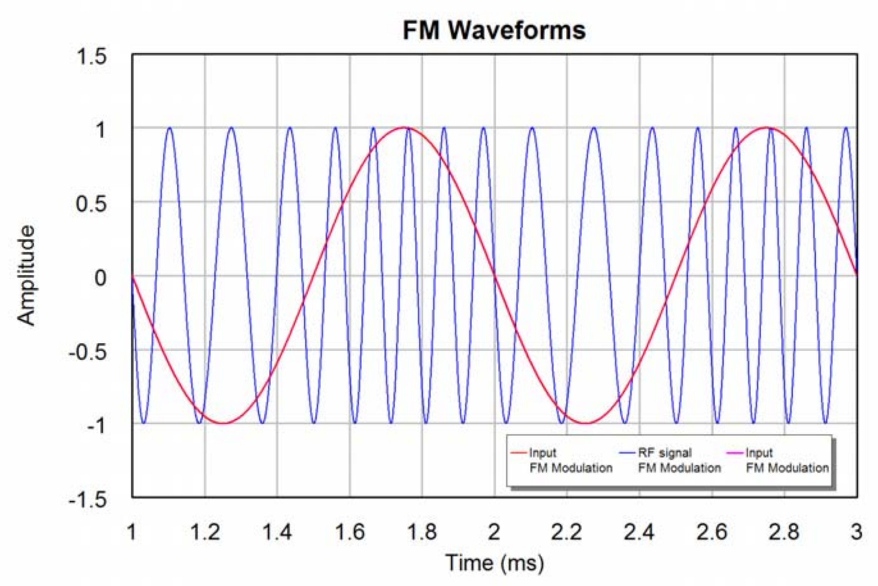


Figure 20. Frequency modulator using an integrator and phase modulator.



Single Tone Modulated FM Spectrum

To gain an understanding of the spectra produced by Frequency modulation, the RF spectrum of a single modulating frequency is determined.

Consider a FM signal produced by a tone modulating signal

$$X_m(t) = A_m \cos(\omega_m t)$$

For this case then

$$Xc(t) = A_c \cos(\omega_c t + k_\omega A_m \int_0^t \cos(\omega_m \gamma)\, d\gamma) \qquad \text{Eqn. 24}$$

$$= A_c \cos(\omega_c t + \frac{k_\omega A_m}{\omega_m} \sin(\omega_m t))$$

$$X_c(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t)) \qquad \text{Eqn. 25}$$

where β is called the modulation index.

$$\beta = \frac{k_\omega A_m}{\omega_m} \quad \text{or} \quad \beta = \frac{k_f A_m}{f_m} \qquad \text{Eqn. 26}$$

where k_f is the voltage to frequency sensitivity of the modulator now in Hz/volt and f_m is the modulating signal frequency in Hz. Often the maximum value of the product $k_f.Am$ is specified for a certain FM modulator and is called the *maximum frequency deviation* or just *frequency deviation* or even just *deviation*. Note this product is the frequency shift in the carrier wave frequency if a dc voltage magnitude of AM was applied to the FM modulator.

For Phase modulation, when a modulating $X_m(t) = A_m \sin(\omega_m t)$ is applied and $\beta = Am\,\phi_d$ equation 25 will also apply. Therefore this analysis applies equally well to PM as to FM.

Since $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$X_{FM}(t) = A_c \cos(\omega_c t).\cos(\beta \sin(\omega_m t)) - A_c \sin(\omega_c t).\sin(\beta \sin(\omega_m t)) \qquad \text{Eqn. 27}$$

There is a standard mathematical expansion for $\cos(\beta \sin \omega_m t)$ and $\sin(\beta \sin \omega_m t)$, which are:

$$\cos(\beta \sin \omega_m t) = Jo(\beta) + \sum_{n\ even}^{\infty} 2\, Jn(\beta) \cos(n\omega_m t) \qquad \text{Eqn. 28}$$

and

$$\sin(\beta \sin \omega_m t) = \sum_{n\ odd}^{\infty} 2\, Jn(\beta) \sin(n\omega_m t) \qquad \text{Eqn. 29}$$

where $Jn(\beta)$ is a *Bessel function of the first kind, and order n.*

Thus

$$X_{FM}(t) = Ac \cos(\omega_c t).[\, Jo(\beta) + \sum_{n\ even}^{\infty} 2\, Jn(\beta) \cos(n\omega_m t)\,]$$

$$- Ac \sin(\omega_c t).[\, \sum_{n\ odd}^{\infty} 2\, Jn(\beta) \sin(n\omega_m t)\,] \qquad \text{Eqn. 30}$$

Now since every cosine product gives rise to an upper and lower sideband pair,

$$X_{FM}(t) = Ac \cos(\omega_c t).\, Jo(\beta) + \sum_{n\ even}^{\infty} [Ac\, Jn(\beta) \cos((\omega_c + n\omega_m)t) + Ac\, Jn(\beta) \cos((\omega_c - n\omega_m)t)\,]$$

$$+ \sum_{n\ odd}^{\infty} [Ac\, Jn(\beta) \cos((\omega_c + n\omega_m)t) - Ac\, Jn(\beta) \cos((\omega_c - n\omega_m)t)\,] \qquad \text{Eqn. 31}$$

The (positive frequency) spectrum of the tone modulated FM signal is therefore like in Figure 21.

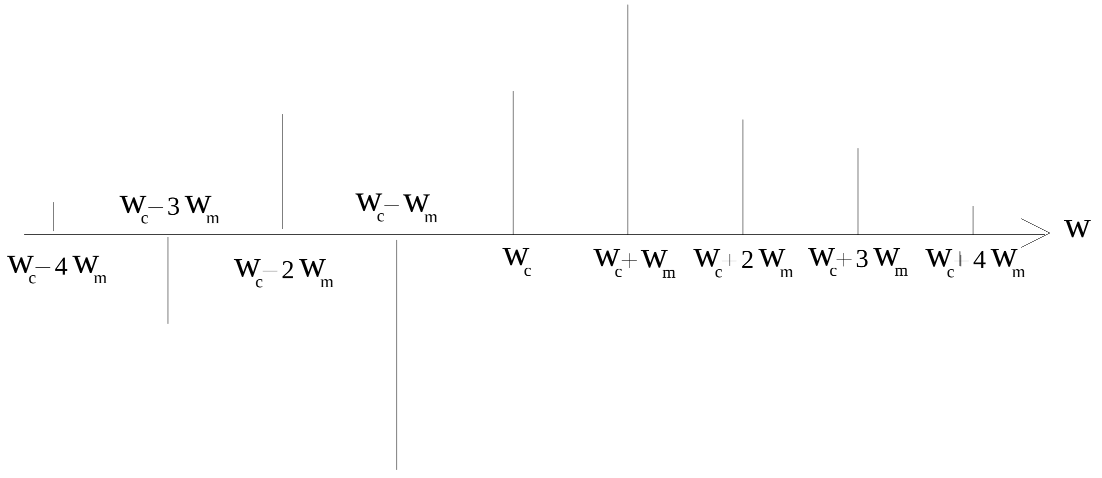


Figure 21.

Note the phase of the odd order lower sideband components are all 180° out of phase with the carrier.

If n is an integer (as it is here) then

$$J_{-n}(\beta) = (-1)^n J_n(\beta)$$

Hence using this relationship we can write equation (21) in a shorter form. That is,

$$X_{FM}(t) = Ac.\sum_{n=-\infty}^{\infty} J_n(\beta)\cos((\omega_c - n\omega_m)t) \qquad \text{Eqn.32}$$

Note:

1. Even for this simple case of tone modulation, the spectrum contains an infinite number of sidebands. Fortunately, only a finite number will be of any significant size (NB as $J_n(\beta)\to 0$ as $n\to\infty$)
2. The carrier component will vary in size depending on the modulating index , β, which in turn depends on the modulating signal magnitude (Am) and frequency (ω_m). Note $\beta = \frac{Am\,k_f}{f_m}$. The carrier component amplitude can be zero since $J_o(\beta)$ can be zero. The first disappearance (corresponding to the first zero of $J_o(\beta)$ occurs when β = 2.4048. This gives a method of finding the FM modulator deviation[1], which equals $A_m.k_f$.
3. The sidebands can disappear depending on Am and f_m.
4. There is NO 1:1 correspondence between the audio spectrum and the FM spectrum.

Bandwidth Requirements

The bandwidth used by a FM signal is equal to $2\times n\times f_m$ where n is the number of significant sidebands above (or below) the carrier frequency. The maximum bandwidth occurs when f_m is maximum. Thus if we have an audio spectrum with a band limit of W Hz say.

$$\text{Bandwidth (max)} = 2\times n\times W$$

We know that n is relate to $\beta = \frac{Am\,k_f}{f_m}$, but what is the relationship?

Define D to be β when we have the maximum deviation ($A_m\,k_f$ is maximum) and f_m = W.

$$\text{ie } D = \frac{\text{Maximum Deviation}}{W}$$

Now from Tables we find n ≈ D+1 and hence we obtain Carson's Rule for bandwidth usage.

$$\text{Maximum Bandwidth} \approx 2\,(D+1)\,W \qquad \text{Eqn. 33}$$

[1] Simply measure fm at the first carrier disappearance and then $Am.k_f = 2.4048 \times f_m$.

For most practical FM modulator though, D is in the range of about 2-10 and this formula tends to underestimate the bandwidth required. In this case, it is often better to use the so called Modified Carson's Rule.

$$\text{Maximum Bandwidth} \approx 2\,(D+2)\,W \qquad \text{Eqn. 34}$$

Equations (23) and (24) give an approximate method of calculating bandwidth usage for an FM system.

Narrow Band FM

It can be seen (from tables) that as the modulation index (β) decreases in an FM system, in general so does the number and size of the sidebands. In the limit, if the modulation index is zero, the signal is not modulated at all and we are left with the carrier.

Figure 22

Narrowband FM (NBFM) is a special case of FM where β is small enough to ensure there are only 2 significant sidebands.

The equation for a tone modulated FM signal is (from (18)),

$$X_{NBFM}(t) = A_c \cos(\omega_c t + \beta \sin(\omega_m t)) \qquad \text{Eqn. 35}$$

or expanding

$$X_{NBFM}(t) = A_c \cos(\omega_c t).\cos(\beta \sin(\omega_m t))$$

$$- A_c \sin(\omega_c t).\sin(\beta \sin(\omega_m t)) \qquad \text{Eqn. 36}$$

Now if β is small (with respect to 1 radian) we can use the following approximations

$$\sin(x) \approx x \text{ and } \cos(x) \approx 1; \text{ x is small.}$$

$$X_{NBFM}(t) = A_c \cos(\omega_c t) - A_c \sin(\omega_c t).\beta \sin(\omega_m t) \qquad \text{Eqn. 37}$$

or

$$X_{NBFM}(t) = Ac \cos(\omega_c t) - \frac{\beta Ac}{2} \cos[(\omega_c - \omega_m)t] + \frac{\beta Ac}{2} \cos[(\omega_c + \omega_m)t] \qquad \text{Eqn. 38}$$

Notes on NBFM

1. Only 2 significant sidebands are used. This signal occupies the same bandwidth as AM
2. Although the spectrum of NBFM looks the same as AM on a spectrum analayser, the 2 sidebands are 180° out-of-phase (ie it is NOT AM).
3. Since β is small (typically <0.2) the amplitudes of the 2 sidebands are also small.
4. Equation (28) gives a means of generating NBFM using multiplier and adder blocks.

Methods of generating and demodulating FM (and PM) will be discussed in lectures.

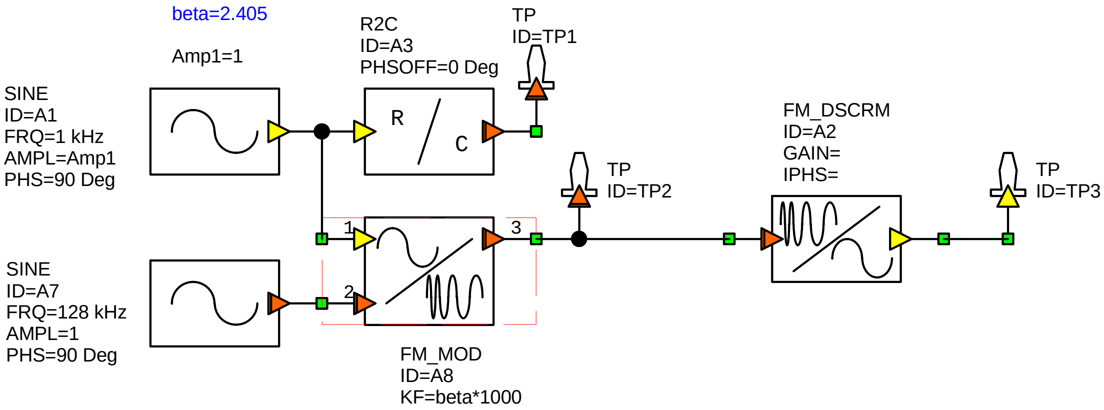


Figure23. VSS block diagram for Frequency Modulation and Demodulation

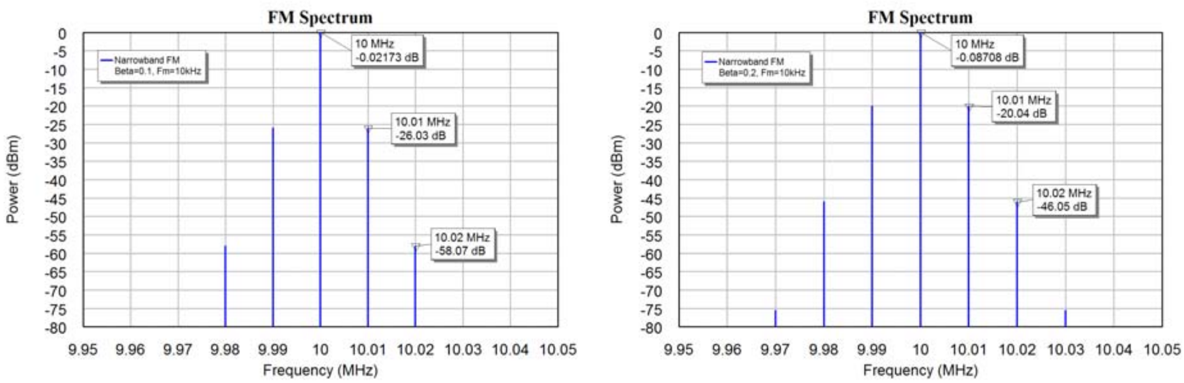


Figure24a. Narrowband FM. β = 0.1 (left) and β = 0.2 (right).

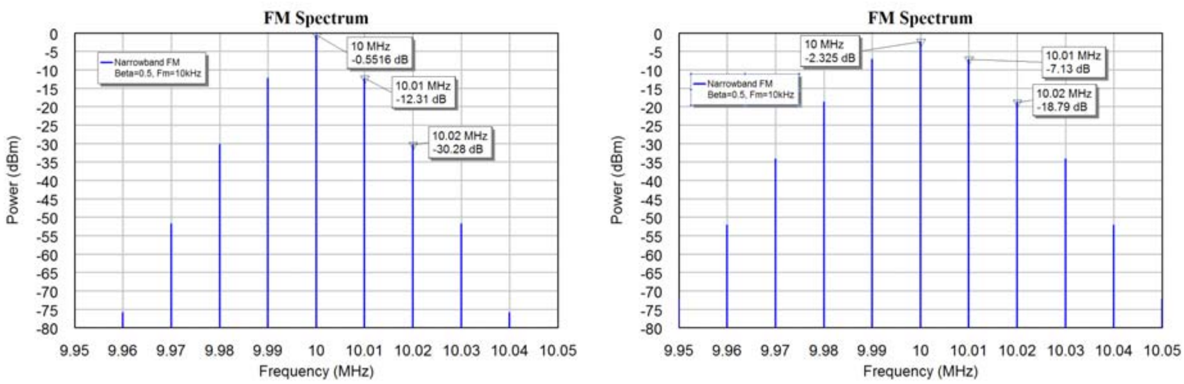


Figure24b. Narrowband FM. β = 0.5 (left) and β = 1.0 (right).

Carrier and sideband disappearance

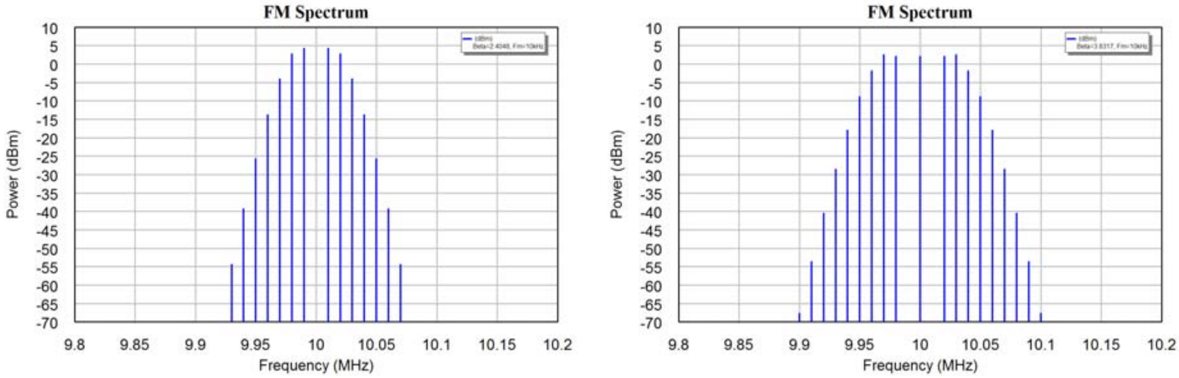


Figure 25a. Carrier and Sideband disappearance. β = 2.4048 (left) and β = 3.8317 (right).

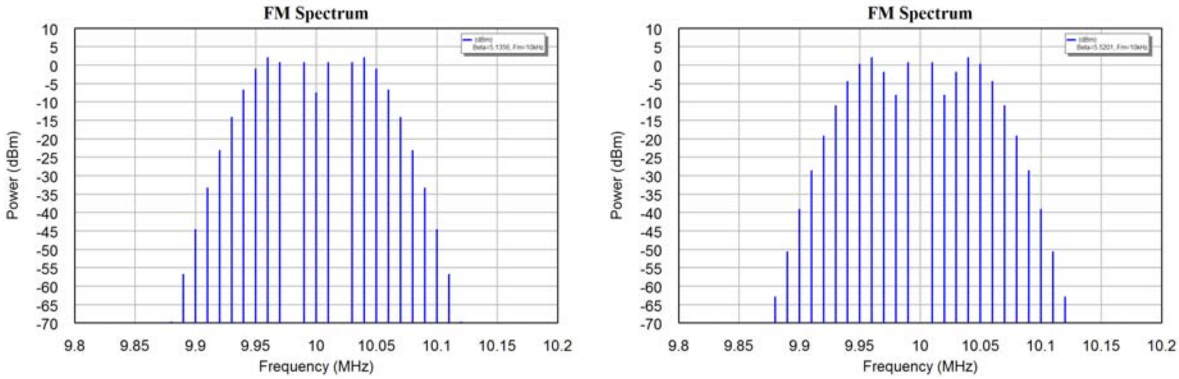


Figure 25b. Carrier and Sideband disappearance. β = 5.1356 (left) and β = 5.5201 (right).

Determination of Bandwidth, Vary F_m, keep F_d fixed.

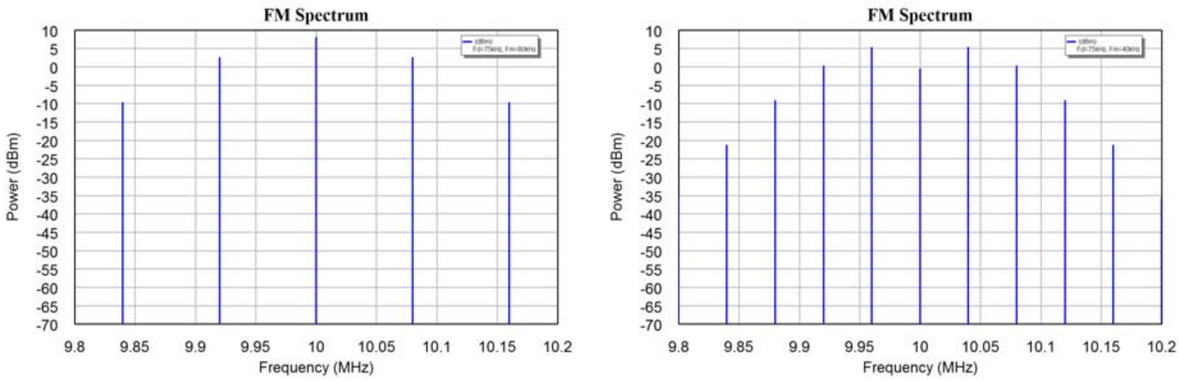


Figure 26a. F_d = 75 kHz, F_m = 80 kHz (left) and F_m = 40 kHz (right).

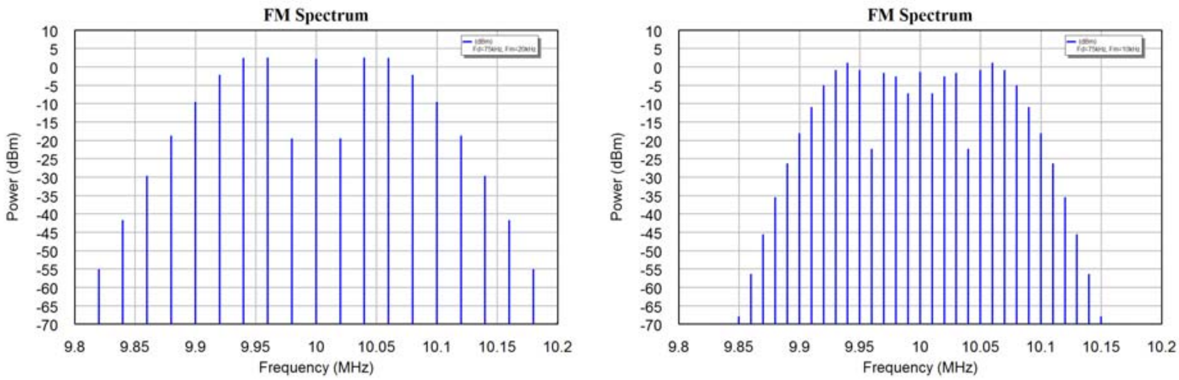


Figure 26b. F_d = 75 kHz, F_m = 20 kHz (left) and F_m = 10 kHz (right).

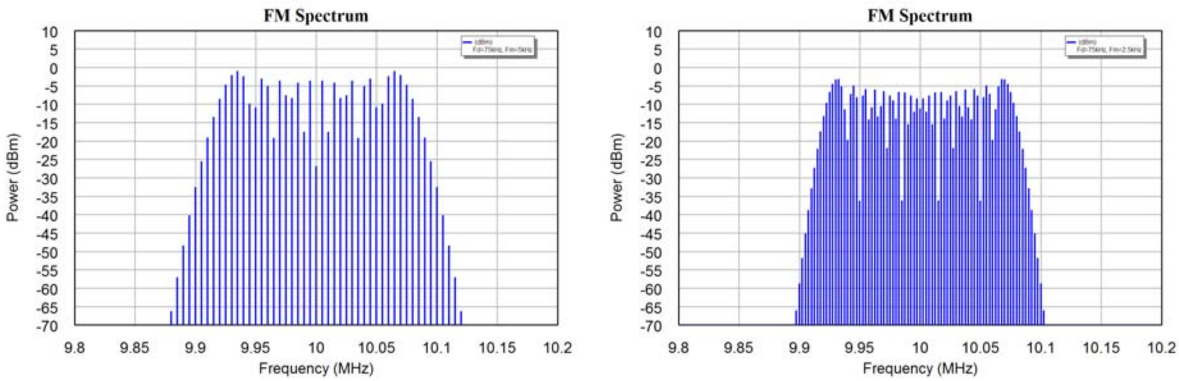


Figure 26c. F_d = 75 kHz, F_m = 5 kHz (left) and F_m = 2.5 kHz (right).

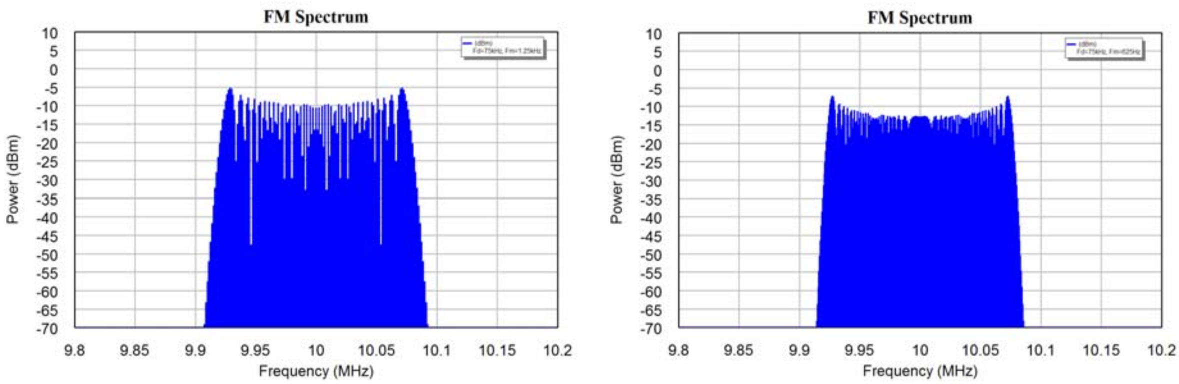


Figure 26d. F_d = 75 kHz, F_m = 1.25 kHz (left) and F_m = 625 Hz (right).

Pre-emphasis and De-emphasis

From considerations of noise in the reception of FM signals, which is outside the scope of this course, it can be shown that the demodulated noise spectral density increases with frequency. This can easily be demonstrated by listening to an FM radio with the muting OFF when it is not tuned to a station. The noise is a high frequency hiss, rather than white noise.

For most program material such as voice or music, the spectral density, falls off with frequency and in general there is not much energy above 3.5 kHz transmitted. 3.5 kHz is the bandwidth for telephony. To improve the signal to noise ratio above approximately 3 kHz, the signal in the transmitter is boosted using a simple operational amplifier network. In the USA and Japan, a 75 µS time constant is used for the pre-emphasis. In Europe and Australia a 50 µS time constant is used, since that prevents overloads due to the higher frequency components that can be present in electronic music.

The Australian pre-emphasis network is shown in figure 27. In the receiver a simple filter as shown in figure 28 is used. The frequency response of these networks is shown in figure 29. It can be seen that if the wrong pre-emphasis is used, a 3 dB treble boost or cut results.

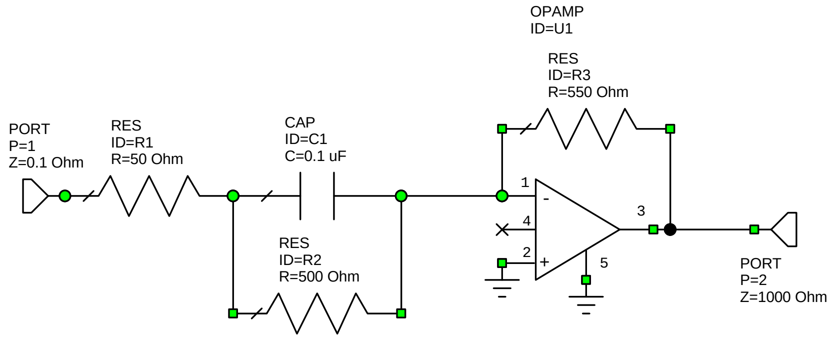


Figure 27. Pre-emphasis network.

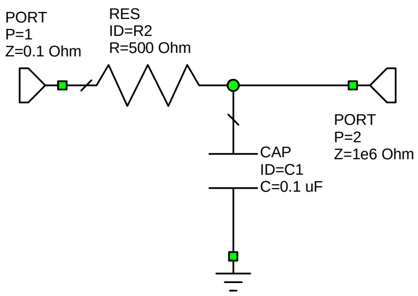


Figure 28. De-emphasis network.

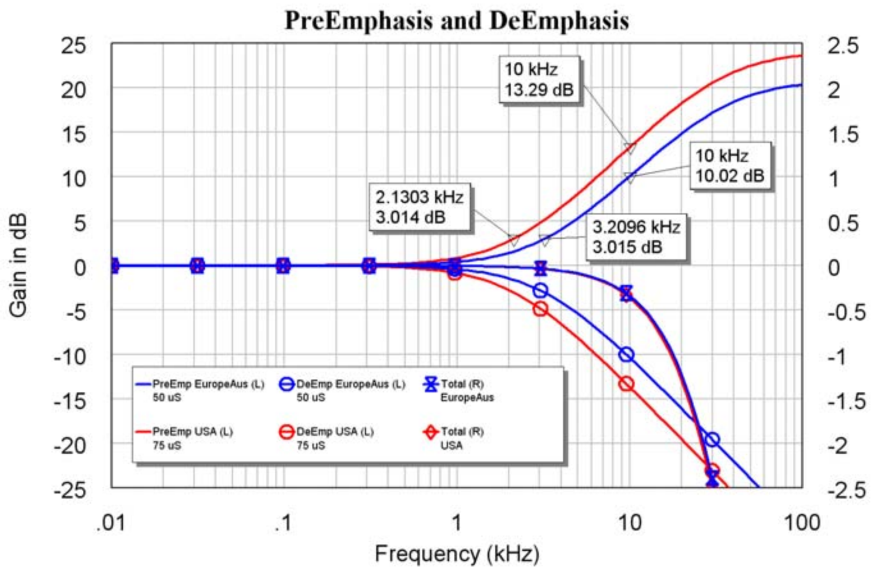


Figure 29. Pre-emphasis and De-emphasis frequency response.

Stereo FM Broadcasting

Mono Specifications

The initial FM broadcasting were mono transmissions. For FM broadcasting the internationally allocated bandwidth is 88 to 108 MHz. The Intermediate frequency (IF) of FM radios is 10.7 MHz. The recommended LO is 10.7 MHz above the signal to be received.

The audio bandwidth is 20 KHz (In practice 15 kHz since most receivers are stereo receivers and will filter out any audio signal above 15 kHz.)

Frequency deviation 75 kHz for FM broadcasting. The PAL TV sound is also an FM system with a 50kHz frequency deviation and a carrier frequency 5.5 MHz above the Video carrier.

Pre-emphasis and De-emphasis 50 micro seconds RC time constant.

The RF bandwidth should be less than 200 kHz. In Australia the minimum frequency spacing between channels in one area is 800 kHz. All channels have a 100 kHz minimum spacing, so tuners use 100 kHz steps.

Stereo System

During the 1950's stereo records were introduced and as a result the FM broadcasts became stereo. The stereo system had to be compatible with the mono system, so that mono receivers would still function correctly.

To achieve this, the mono = (L+R)/2 signal is transmitted normally. The stereo = (L_R)/2 signal is modulated on a 38 kHz Double Sideband Suppressed Carrier system. In order to be able to synchronously demodulate the stereo signal, a pilot at 19 kHz, i.e. half the 38 kHz and a frequency deviation of 0.1 kHz is added. In the receiver this is used to generate the 38 kHz subcarrier for demodulation of the stereo component. In order to ensure that the 75 kHz allowable frequency deviation is not exceeded, the peak amplitudes of the Left and Right and thus the maximum amplitudes of the Mono and Stereo signals are reduced to 67.5 kHz deviation. Pre-emphasis is applied to the Left and Right signals before these are coded in the stereo coder and the corresponding de-emphasis is applied after the stereo decoding. The block diagram of the stereo coder and decoder is shown in figure 30.

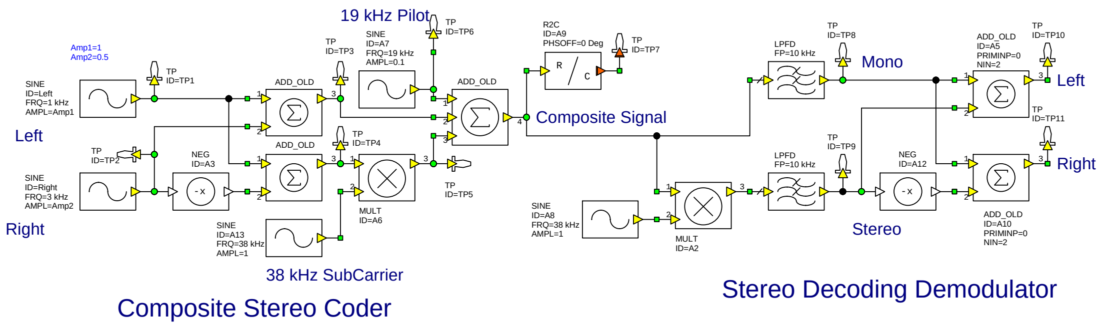


Figure 30. Block diagram of stereo coder.

Figure 31 shows the waveforms in the stereo coder when the pilot signal is disabled and the left input signal is 1kHz with an amplitude of 1 and the right input signal is 1 kHz with an amplitude of -0.5.

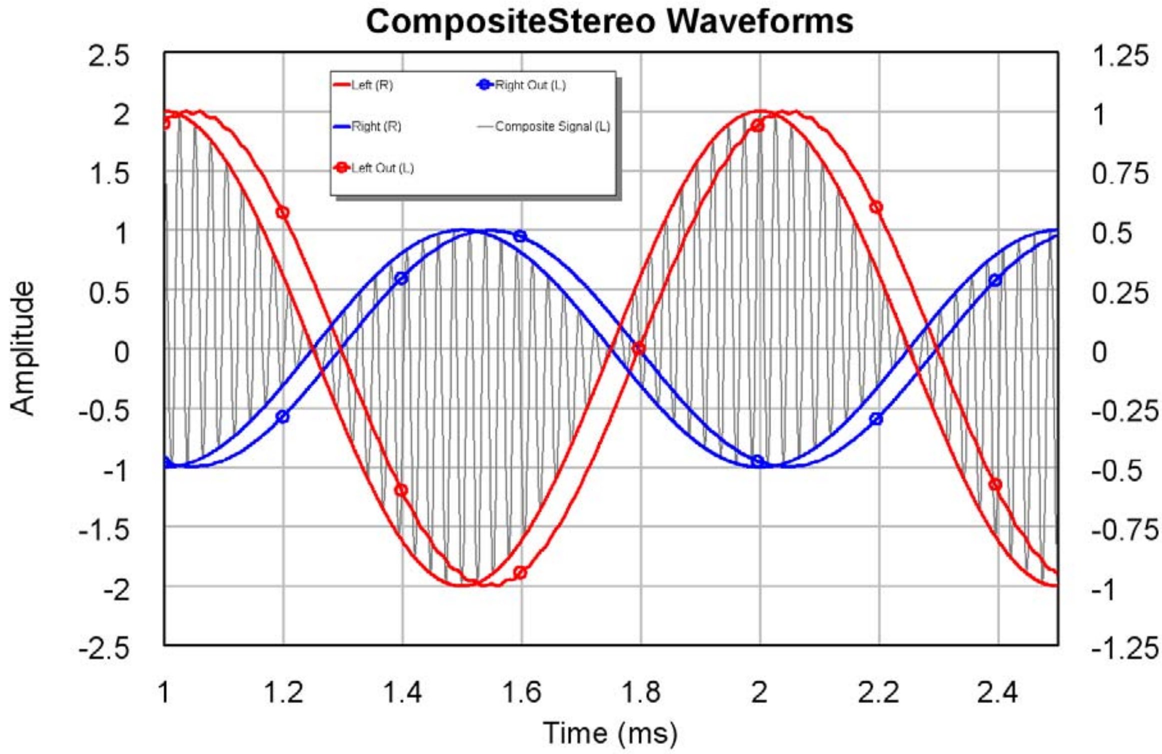


Figure 31. Stereo coder waveforms

When the frequency of the right input signal is changed to 3 kHz, the spectrum shown in figure 32 results. The left and right input frequencies match those of the Mono signal, with the only difference being the amplitude. The double sideband suppressed carrier components of the stereo signals and the 19 kHz pilot signal can be clearly seen.

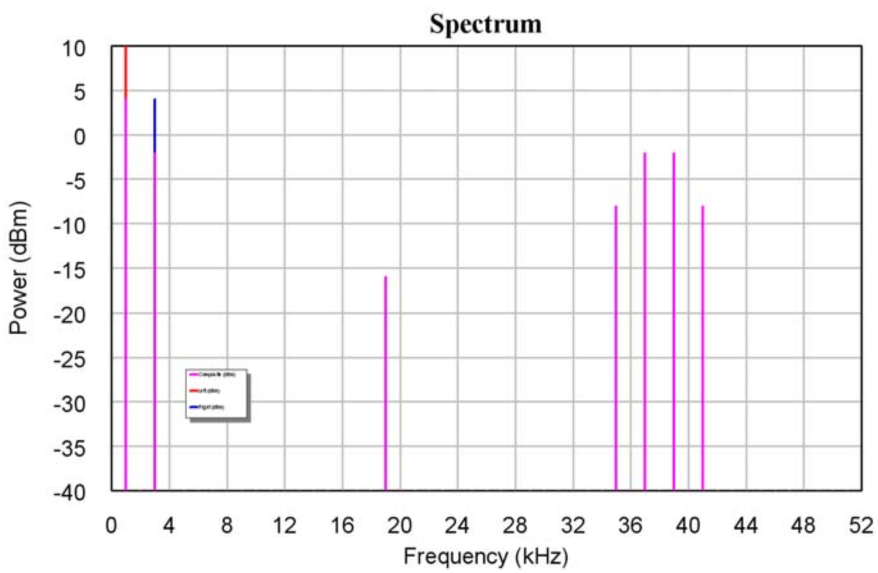


Figure 32. Stereo coder spectra.

Additional Signals

The left and right signals are restricted to a 15 KHz upper frequency, which gives a clear spectrum between 15 kHz and 23 kHz is clear apart from the pilot signal. In Europe, the spectrum around 16 kHz is used to transmit program information, like what radio station is being received, the type of program, Classical Music, Rock, Talkback etc. For a car radio, when the signal becomes out of range, it will automatically search for the same station (like BBC1) or the same type of program if the same station is not available. In Norway the frequency band above 52 kHz (38 kHz + 15 kHz) is used for paging, in the Australia a 67 kHz and sometimes 92 kHz subcarrier can be used for supplementary transmissions. In the USA this is called SCA (Subsidiary Communications Authority) in Australia this is ACS (Ancillary Communications Service) and can be used for data or program services. These systems are not widely used and can cause crosstalk between this service and the main program on poorer receivers. With Digital Audio Broadcasting being introduced in the near future, there is little interest in Australia for this service.

FM Generation

Varacter Diode Oscillator (VCO)

Direct Digital Synthesis

In Direct Digital Synthersis, a counter is incremented at each sampling period. That counter contains the phase and used as input to a lookup table to convert that phase to a sine output. The output from that sine lookup table is used to drive a DAC to produce the required sinewave output. DDS IC’s are available at up to 1Gsps output rate and 14 bit DAC output. (AD9910).

FUNCTIONAL BLOCK DIAGRAM

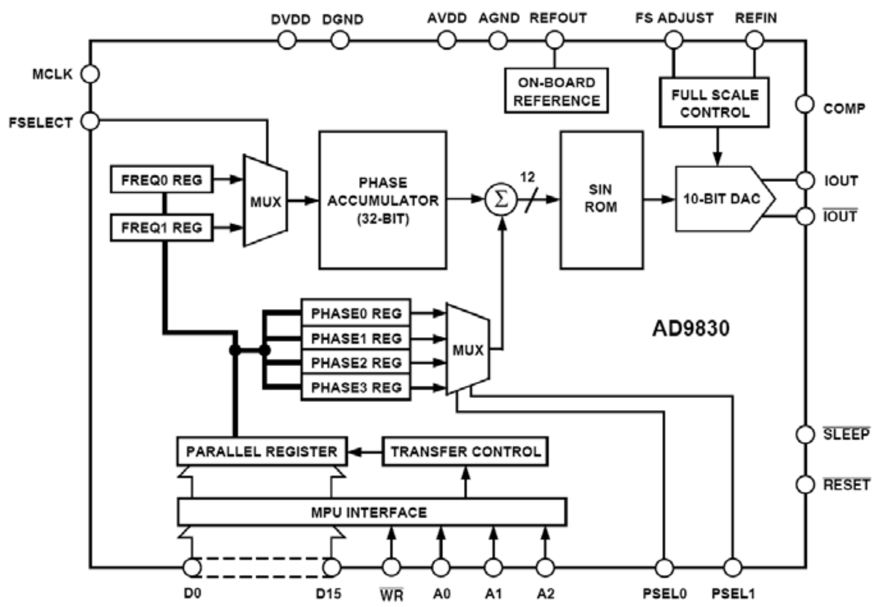


Figure 34. Typical DDS Block diagram. [Analog Devices AD9830 Data Sheet].

Incrementing the phase counter by different amounts at each clock pulse changes the operating frequency in a linear manner. i.e. doubling the phase accumulator step size doubles the output frequency. This allows the frequency of the output waveform to be digitally controlled. If that digital control is due to an audio signal then Frequency Modulation results. Since the phase step is linearly related to the frequency, controlling the phase step directly by the modulating signal results in an inherently linear Frequency Modulation, without any microphony. However the DAC will produce some spurii, which will typically be more than 60 dB below the carrier and thus be within the specifications for FM transmitters. In addition to FM modulation, the DDS can be used as a phase modulator by the appropriate control of the phase step size or by adding phase offsets after the phase accumulator as shown in figure 34. DDS devices are now used in some commercial FM transmitters.

FM demodulation

The Quadrature Detector is the most commonly used FM discriminator used in FM radios. It consists of a resonant network (L1 and C1) being fed by a high impedance (L2). The frequency response is shown in figure 36. The network is used as a frequency to phase converter and an exclusive OR gate or a multiplier (not shown) is used as a phase detector, resulting in an output voltage that is proportional to frequency.

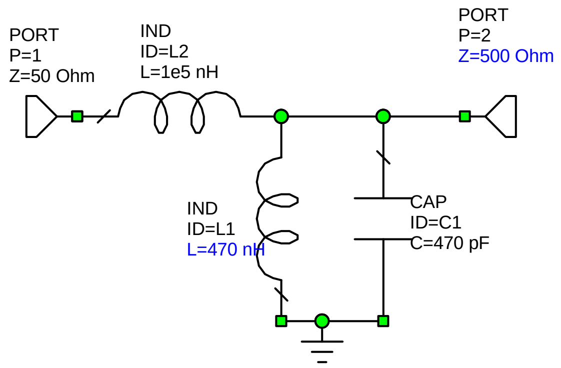


Figure 35. Quadrature Detector Circuit.

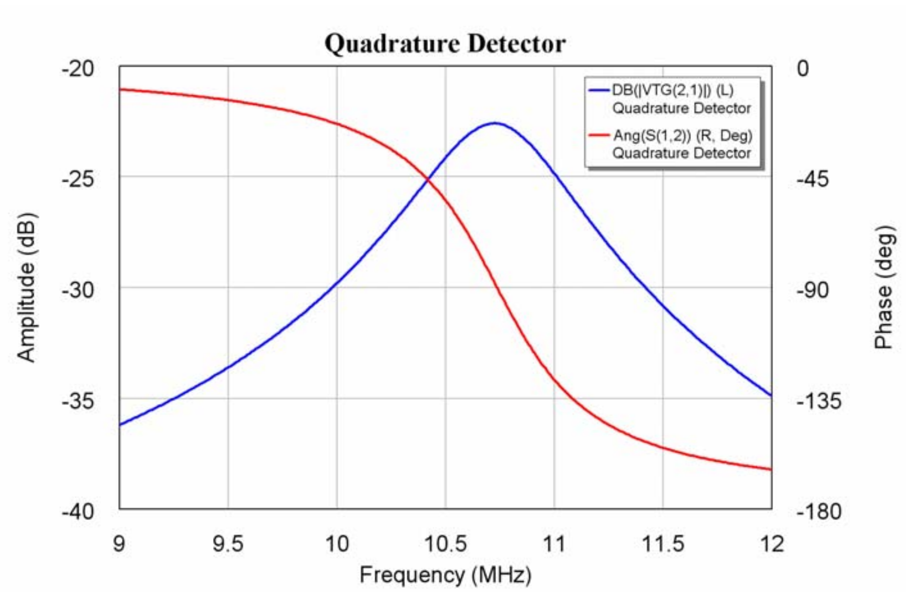


Figure 36. Quadrature Detector Frequency Response.

Stereo AM

Super Heterodyne Receiver

AM, FM, Pal TV

Trends in Receivers for AM and FM radio