Pulsed Modulation Systems

Pulse Amplitude Modulation

A Rectangular Pulse in the time domain is:

F(t) = 0 $t < -\tau/2$
F(t) = A $-\tau/2 < t < \tau/2$
F(t) = 0 $t > \tau/2$

Repeating every T_0 seconds

In the Frequency Domain:

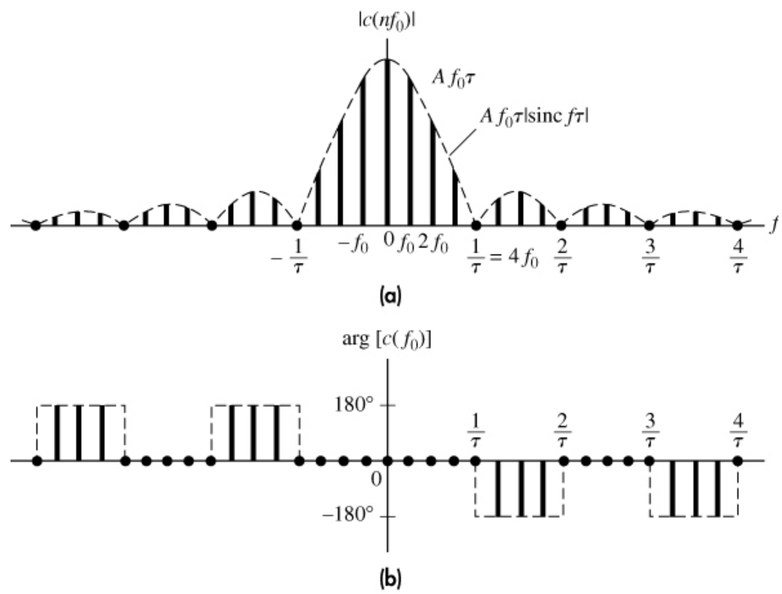
$$C_n = \frac{1}{T_0}\int_{-\tau/2}^{\tau/2} A e^{-j2\pi n f_0 t} dt = \frac{A}{-j2\pi n f_0 T_0}(e^{-j\pi n f_0 \tau} - e^{j\pi n f_0 \tau})$$

$$C_n = \frac{A\tau}{T_0}\frac{Sin(\pi n f_0 \tau)}{\pi n f_0 \tau}$$

From Fourier Series:

$$F(f) = \sum_{n=-\infty}^{n=\infty} C_n e^{-jn2\pi f_0 t} = \sum_{n=-\infty}^{n=\infty} \frac{A\tau}{T_0}\frac{Sin(\pi n f_0 \tau)}{\pi n f_0 \tau} e^{-jn2\pi f_0 t}$$

The spectrum has components spaced at $1/T_0$ and an envelope given by the Sinx/x curve.

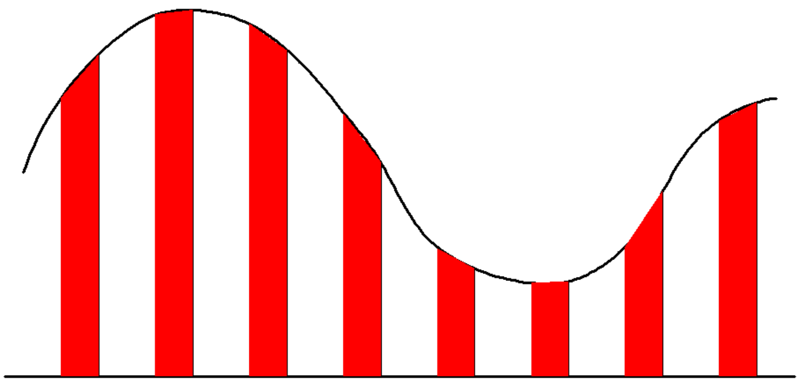


Samping Pulse Spectrum (From Carlson, Fig 2.1-8)

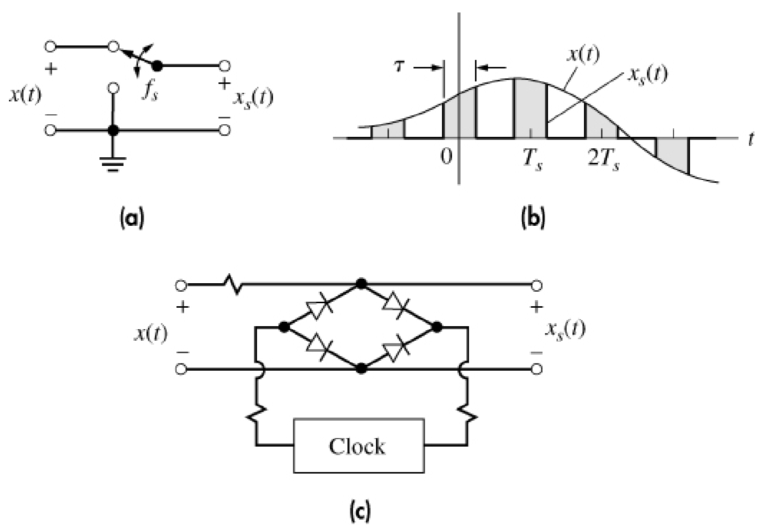
In ideal sampling the pulse width is zero, i.e. τ=0. Under those conditions the zeros in the Sinx/x spectral envelope becomes very high in frequency, so that all the DSB components have the same amplitude. (See Haykin Fig 3.2).

For Practical sampling the pulse with τ is finite and the Sinx/x curve is evident. If τ=T/3, then the third harmonic of the sampling waveform disappears.

A



Hardware for a practical sampler is shown below:

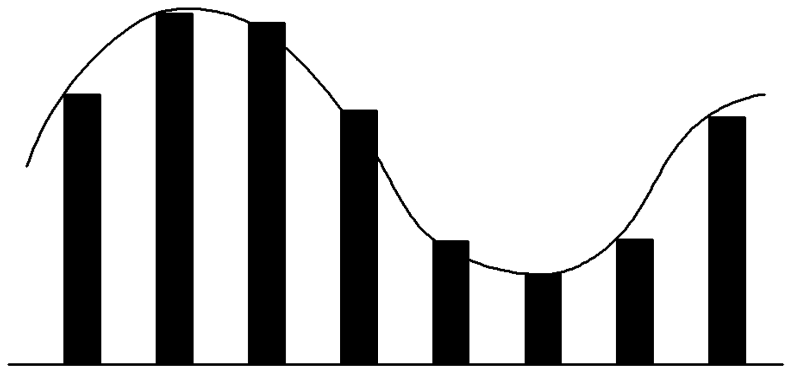


(From Carlson Communication⇔ Systems)

Note to prevent aliasing the sampling frequency must be > twice the input signal Bandwidth. The limit F_s=2W is called the Nyquist Sampling rate.

Sample and Hold

To recover a sampled signal, such as is done with a Digital to Analog converter, the output is held constant for a certain time, usually until the next value becomes available.



The transfer function of the Sample and Hold circuit can be seen as a circuit with an impulse response of (u(t)-u(t-T0), i.e a unit impulse followed by a negative unit impulse a time τ later.

The transfer function of that (using the Laplace transform) is:

$$H(s) = \frac{1}{s} - \frac{1}{s}e^{-s\tau} = \frac{1}{s}e^{-s\tau/2}(e^{s\tau/2} - e^{-s\tau/2})$$

Putting S=jω to get the frequency response gives:

$$H(\omega) = \tau\, e^{-j\omega\tau/2}\,\frac{\dfrac{(e^{j\omega\tau/2} - e^{-j\omega\tau/2})}{2j}}{\omega\tau/2} = \tau\, e^{-j\omega\tau/2}\,\frac{Sin(\omega\tau/2)}{\omega\tau/2}$$

The sample and hold operation is thus like a filter, which has a phase shift of $\omega\tau/2$ and a zero when at $\omega\tau/2 = \pi$. Putting $\omega=2\pi f$, results in the zero occurring when $f=1/\tau$. When $\tau=T_s$ then the null will occur at the sampling frequency. This is called the **Aperture Effect.**

In addition at half the sampling frequency the attenuation is $Sin(\pi)/\pi = 1/\pi = 9.9$ dB, so that a significant attenuation results at high frequencies. Sometimes this attenuation is compensated for using a digital filter, with the inverse amplitude response of the aperture effect.

Pulse Width Modulation

Pulse Position Modulation