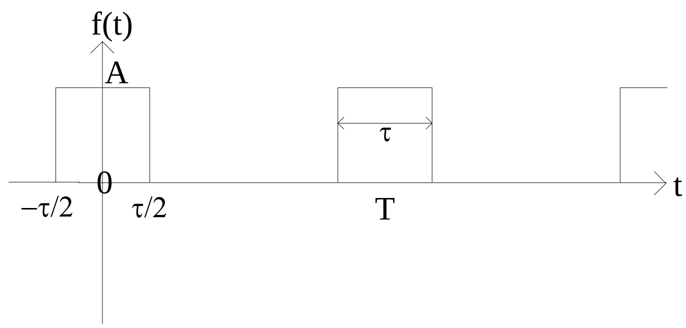
EE3700 : Communications I

Tutorial 1- Solution

1. A certain periodic band-limited signal consists of frequencies at dc, 500Hz and at 1kHz. The signal can be written as
   $$s(t) = 12 + 20 \cos 1000\pi t \; - \; 20 \sin 1000\pi t \; + \; 12 \cos 2000\pi t \; - \; 6 \sin 2000\pi t$$
   Express this signal as a Fourier series in complex exponential form.

2. Determine the Fourier coefficients C_n for the following periodic signal.



3. Considering the waveform in question 2, estimate the amount of the original signals power which would pass through a filter that rejects harmonics above 2^{nd} order (ie greater than $2\times f_0 = 2/T$). Assume T=1mS, A = 10 volts and τ = 0.25mS.

4. A radar system transmits pulses of 200nS duration. Estimate the bandwidth of the transmitter circuitry necessary to ensure that at least 90% of the energy of the pulse is transmitted.

No1. The signal consists of cos and sin terms. Lets examine the complex Fourier series representation of these signals then.

COS TERMS. $s(t) = A\cos W_o t$.

$$C_n = \frac{A}{T}\int_0^T \cos W_o t \; e^{-jnW_o t}\, dt \quad ; n = 1, 2, 3 \ldots$$

$$= \frac{A}{T}\int_0^T \cos W_o t \left[\cos nW_o t - j\sin nW_o t\right] dt.$$

$$= \frac{A}{2T}\int_0^T \cos\left[W_o t - nW_o t\right] + \cos\left[W_o t + nW_o t\right]$$

$$- j\left\{\sin\left(W_o t + nW_o t\right) - \sin\left(W_o t - nW_o t\right)\right\} dt.$$

Note as all terms are sinusoidal with a frequency which is a multiple of W_o, the integration will be zero, except when $n = \pm 1$.

$$\therefore \quad C_1 = \frac{A}{2} \quad \& \quad C_{-1} = \frac{A}{2}.$$

AND $$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnW_o t}.$$

$$= \frac{A}{2} e^{jW_o t} + \frac{A}{2} e^{-jW_o t}$$

SIN TERMS $\quad s(t) = A \sin W_o t.$

$$C_n = \frac{A}{T}\int_0^T \sin W_o t \left[\cos nW_o t - j\sin nW_o t\right] dt.$$

$$= \frac{A}{2T}\int_0^t \sin W_o t(1+n) + \sin W_o t(1-n)$$

$$-j\left\{\cos W_o t(1-n) - \cos W_o t(1+n)\right\} dt.$$

Again all terms, except where $n = \pm 1$, will be zero.

$$C_1 = -j\frac{A}{2} \qquad C_{-1} = +j\frac{A}{2}.$$

$$s(t) = -j\frac{A}{2}e^{j\omega_0 t} + j\frac{A}{2}e^{-j\omega_0 t}.$$

In question 1). $s(t) = 12 + 20\cos 1000\pi t - 20\sin 1000\pi t + 12\cos 2000\pi t - 6\sin 2000\pi t$.

Set $\omega_0 = 1000\pi$. so.

$$s(t) = 12 + 20\cos\omega_0 t - 20\sin\omega_0 t + 12\cos 2\omega_0 t - 6\sin 2\omega_0 t$$

$C_0 = \frac{1}{T}\int_0^T s(t)\,dt$ is the AVERAGE of $s(t)$

$= 12.$

$$\therefore\ s(t) = 12 + \frac{20}{2}e^{j\omega_0 t} + \frac{20}{2}e^{-j\omega_0 t} + \frac{20j}{2}e^{j\omega_0 t} - \frac{20j}{2}e^{-j\omega_0 t}$$
$$+ \frac{12}{2}e^{j2\omega_0 t} + \frac{12}{2}e^{-j2\omega_0 t} + \frac{6j}{2}e^{j2\omega_0 t} - \frac{6j}{2}e^{-j2\omega_0 t}$$

or
$$s(t) = 12 + 10e^{j\omega_0 t} + 10e^{-j\omega_0 t} + j10e^{j\omega_0 t} - j10e^{-j\omega_0 t}$$
$$+ 6e^{j2\omega_0 t} + 6e^{-j2\omega_0 t} + j3e^{j2\omega_0 t} - j3e^{-j2\omega_0 t}.$$

OR

$$s(t) = 12 + (10+j10)e^{j\omega_0 t} + (10-j10)e^{-j\omega_0 t} + (6+j3)e^{j2\omega_0 t} + (6-j3)e^{-j2\omega_0 t}$$

As a line spectra the signal would have spikes at dc, ω_0 and $2\omega_0$. The magnitudes are given from the expression above. eg. At ω_0 $|C1| = \sqrt{200}$

No 2. These can be a bit tricky! The integration can be difficult.

We can integrate our function over any period. T. In this case lets integrate from $-T/2$ to $T/2$.

$$\therefore \quad C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt.$$

$$= \frac{1}{T}\int_{-\tau/2}^{\tau/2} A \; e^{-jn\omega_0 t}\, dt.$$

$$= \frac{A}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-\tau/2}^{\tau/2} = \frac{A}{-jn\omega_0 T}\left[e^{-jn\omega_0\tau/2} - e^{jn\omega_0\tau/2}\right]$$

$$= \frac{2A}{-n\omega_0 T}\left[\frac{e^{j(-n\omega_0\tau/2)} - e^{-j(-n\omega_0\tau/2)}}{2j}\right]$$

$$C_n = \frac{2A}{+n\omega_0 T}\sin\left(+n\omega_0\tau/2\right)$$

$$C_0 = \text{average Voltage} = A.\frac{\tau}{T}.$$

No3 Firstly, calculate the power of the signals from their time response. When talking about power we need to use RMS. values of voltage though!

That is; POWER = $[V_{RMS}]^2$.

where $V_{RMS} = \sqrt{\overline{(f(t))^2}}$ { is the square root of the the mean of f(t) squared.

b) $V_{RMS} = \sqrt{\frac{1}{T}\int_{-\tau/2}^{\tau/2} A^2 \, dt.} = A\sqrt{\frac{\tau}{T}}$

If. $T = 1mS$, $A = 10V$ + $\tau = 0.25mS$.

$V_{RMS} = 10 \cdot \frac{1}{2} = 5V$

Power = 25 watts.

If we filter the signals so that only the first 2 harmonics of the original signal are allowed to pass.

Filtered power = $\sum_{n=-2}^{n=2} |Cn|^2$

If all the power were to pass, the sum would be from n = -∞ to +∞.

Here $C_0 = A\frac{\tau}{T}$ + $C_n = \frac{2A}{n\omega_0 T} \sin(n\omega_0\tau/2)$

If. $A = 10$, $\tau = 0.25\,mS$, $T = 1\,mS$, $\omega_0 = 6.283 \times 10^3$ rad/sec

$\underline{n=0}$ $C_0 = 2.5$

$\underline{n=1}$ $C_1 = \dfrac{2 \times 10}{6.283 \times 10^3 \times 1 \times 10^{-3}} \sin\left[\dfrac{6.28 \times 10^3 \times 0.25 \times 10^{-3}}{2}\right]$

$= 3.183 \times \sin(0.785) = 3.183 \times 0.7068$

$= 2.25$

$\underline{n=2}$ $C_2 = \dfrac{2A}{2\omega_0 T} \cdot \sin\left(2\ \omega_0 \tau/2\right)$

$= \dfrac{3.183}{2} \cdot \sin(2 \times 0.785) = 1.591 \times 1.0.$

$= 1.591$

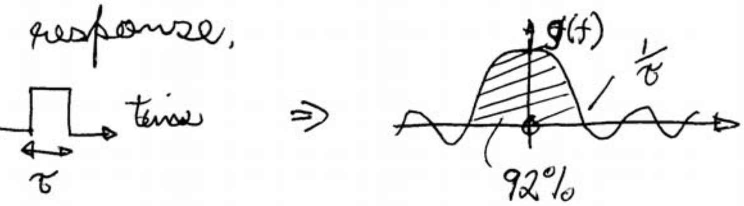
$\therefore$ FILTERED POWER. $= |C_{-2}|^2 + |C_1|^2 + |C_0|^2 + |C_1|^2 + |C_2|^2$

$= 2.53 + 5.0625 + 6.25 + 5.0625 + 2.53$

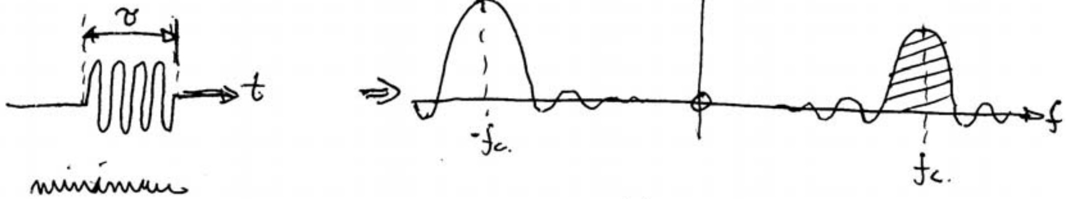
$= \underline{21.43 \text{ Watts.}}$

Note in this case <u>most</u> of the original signals power (25W) will pass through the filter. This filter is therefore reasonably efficient in this situation.

No 4 We showed in lecture notes that about 92% of the power contained in a square pulse occurs in the main lobe of the frequency response.

ie 

In a radar system a pulse of high frequency carrier signal is transmitted. The frequency response is thus the same as a pulse but shifted to around the carrier frequency.

ie 

The minimum bandwidth of the transmitter to ensure 90% power transmission is thus $\approx 2 \times \frac{1}{\tau}$.

$$\underset{(B)}{\text{Bandwidth}} \geq 2 \times \frac{1}{\tau}$$

In this case $B \geq 2 \times \frac{1}{200 \times 10^{-9}} = 10\ \text{MHz}$.

Note a large bandwidth is needed to transmit thin pulses.