EE3700 – Communications I

Tutorial 2 - Solution

1. Figure 1 shows the waveform for a copper-modulated sine wave. This modulated wave is applied to a demodulator consisting of a mixer and an ideal low-pass filter with a pass band amplitude response of 1 and a bandwidth of B Hz. The LO input of the mixer is supplied with a locally generated carrier signal 10 cos(2πft + ϕ) volts.
   a) Find the spectrum at the output of the mixer assuming ϕ=0.
   b) If f = 100kHz, ϕ=0 and modulation frequency f_m is in the range from 10Hz to 1kHz, specify the permissible range of the filter bandwidth B to allow only the modulating signal at the filter output.

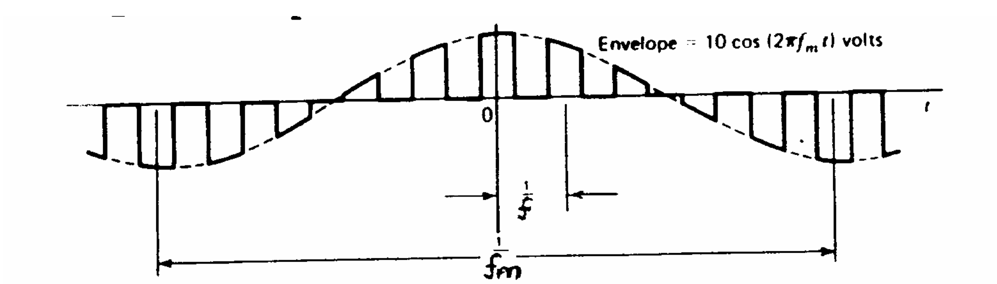


Figure 1

2. A Double SideBand (DSB) signal is demodulated by applying it to a synchronous detector. Evaluate the effect of a frequency error Δf in the local carrier frequency of the receiver, measured with respect to the actual carrier frequency of the DSB signal. For the case of a tone modulated wave, show that because of the frequency error, the demodulated wave will exhibit “beats” at the error frequency. Use a sketch to demonstrate the effect.

3. One circuit that can be used to synchronously demodulate a DSB signal is the Costa receiver. The block diagram of this circuit is shown in Figure 2. Describe how this circuit works.

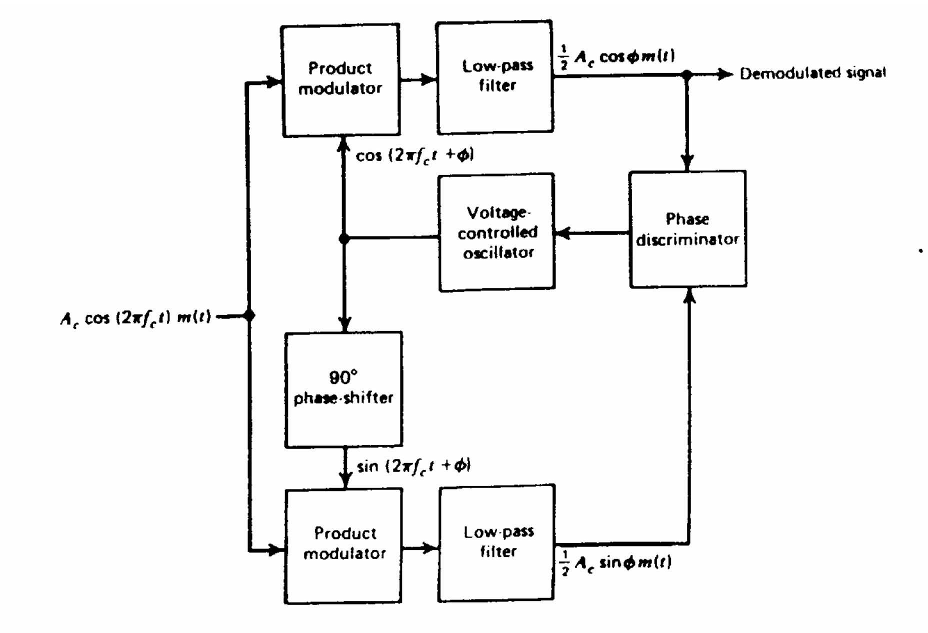
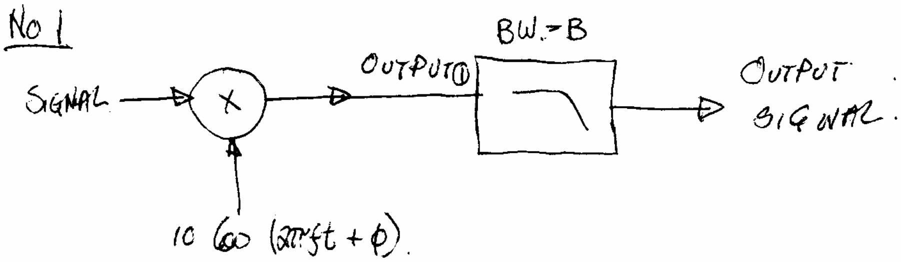

Product modulator
Low-pass filter
$\frac{1}{2} A_c \cos\phi\, m(t)$
Demodulated signal
$\cos(2\pi f_c t + \phi)$
Voltage-controlled oscillator
Phase discriminator
$A_c \cos(2\pi f_c t)\, m(t)$
90° phase-shifter
$\sin(2\pi f_c t + \phi)$
Product modulator
Low-pass filter
$\frac{1}{2} A_c \sin\phi\, m(t)$

Figure 2

EE 3702 TUTORIAL. 2.

No 1



The signal is just a square wave multiplied by $10 \cos(2\pi f_m t)$.

Thus Signal = SQUARE WAVE × $10 \cos(2\pi f_m t)$.

The SQUARE WAVE can be represented by its fourier series.

i.e. $$S.W = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f t(2n-1)\right] + 1/2$$

The FOURIER SERIES OF THE SIGNAL IS THUS
$$\frac{20}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi t(f(2n-1) + f_m)\right] + \cos\left[2\pi t\,(f(2n-1) - f_m)\right]$$
$$+ \cos(f_m)$$

The MIXER multiplies this signal with a second cos term, $10 \cos(2\pi f t)$.

Thus OUTPUT ①
$$= \frac{100}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi t(f.2n + f_m)\right] + \cos\left[2\pi t \cdot (f(2n-2) + f_m)\right]$$

$$+ \cos\left[2\pi t\left(f\,2n - f_m\right)\right] + \cos\left[2\pi t\left(f(2n-2) - f_m\right)\right]$$
$$+ \cos\left[2\pi t\left(f + f_m\right)\right] + \cos\left[2\pi t\left(f - f_m\right)\right]\Big]$$

If we look at this signal we have AM components at 3 carrier frequencies.

$f_{c1} = f.2n \;;\quad n = 1, 2 \ldots \infty$

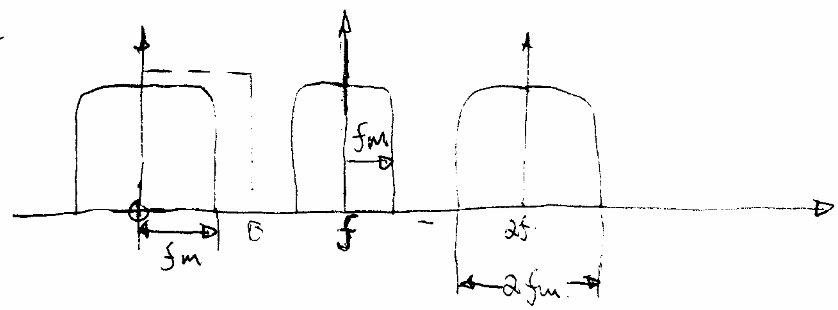
$f_{c2} = f(2n-2) \;;\quad n = 1, 2 \ldots \infty$

$f_{c3} = f.$

When $f_{c2} = 0$; $n = 1$ we have recovered the audio.

The "others" components occurring at f, 2f, 4f, 6f etc are unwanted and must be filtered out.

HAVE



If $f = 100$ KHz, f_m ; 10Hz → 1 KHz

We can see that $B > 1$ KHz.

and $B < 99$ KHz.

ie.

$$1\,\text{KHz} < B < 99\,\text{KHz}.$$

No 2

$$DSBSC. - S(t) = A_c \cos(2\pi f_c t) . m(t)$$

Demodulator frequency. $D(t) = \cos(2\pi(f_c + \Delta f)t)$

$$\text{DEMODULATED SIGNAL} = S(t) D(t) = m(t) \left[\cos(2\pi t(\Delta f)) + \cos(2\pi t(2f_c + \Delta f)) \right]$$

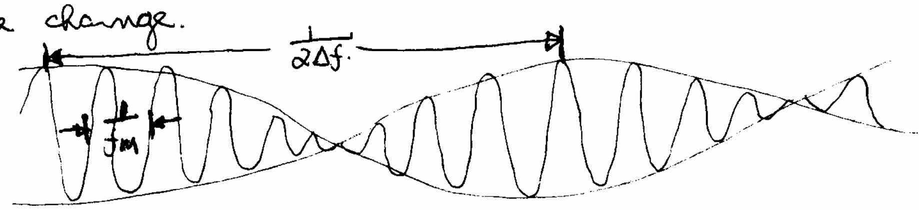
The Demodulator also contains a filter which removes the higher frequency components.

$$\Rightarrow \quad \text{DEMOD. SIG} = m(t) . \cos(2\pi t(\Delta f))$$

If the modulating wave is a sinusoid say $\cos(2\pi f_m t)$.

$$\text{Then DEMOD. SIG.} = \cos(2\pi t(f_m + \Delta f)) + \cos(2\pi t(f_m - \Delta f))$$

This is a DSB signal, Usually Δf would be small, f_m is constant. Beats occurs as the amplitude and phase change.



No 3

Costas Receiver

One method of obtaining a practical synchronous receiver system, suitable for demodulating DSB-SC waves, is to use the *Costas receiver*[3] shown in Fig. 3.15. This receiver consists of two coherent detectors supplied with the same input signal, namely, the incoming DSB-SC wave $A_c \cos(2\pi f_c t)\, m(t)$, but with individual local oscillator signals that are in phase quadrature with respect to each other. The frequency of the local oscillator is adjusted to be the same as the carrier frequency f_c, which is assumed known *a priori*. The detector in the upper path is referred to as the *in-phase coherent detector* or *I-channel,* and that in the lower path is referred to as the *quadrature-phase coherent detector* or *Q-channel.* These two detectors are coupled together to form a *negative feedback* system designed in such a way as to maintain the local oscillator synchronous with the carrier wave.

To understand the operation of this receiver, suppose that the local oscillator signal is of the same phase as the carrier wave $A_c \cos(2\pi f_c t)$ used to generate the incoming DSB-SC wave. Under these conditions, we find that the *I*-channel output contains the desired demodulated signal $m(t)$, whereas the *Q*-channel output is zero due to the quadrature null effect of the *Q*-channel. Suppose next that the

local oscillator phase drifts from its proper value by a small angle ϕ radians. The *I*-channel output will remain essentially unchanged, but there will now be some signal appearing at the *Q*-channel output, which is proportional to $\sin\phi \simeq \phi$ for small ϕ. This *Q*-channel output will have the same polarity as the *I*-channel output for one direction of local oscillator phase drift and opposite polarity for the opposite direction of local oscillator phase drift. Thus, by combining the *I*- and *Q*-channel outputs in a *phase discriminator* (which consists of a multiplier followed by a low-pass filter), as shown in Fig. 3.15, a dc control signal is obtained that automatically corrects for local phase errors in the *voltage-controlled oscillator.*

It is apparent that phase control in the Costas receiver ceases with modulation and that phase-lock has to be reestablished with the reappearance of modulation. This is not a serious problem when receiving voice transmission, because the lock-up process normally occurs so rapidly that no distortion is perceptible.