EE3700 - Communication I

Tutorial 3 - Solution

1. The transmitter stage of a modem uses a carrier frequency of 1500Hz. During testing, the carrier is frequency modulated by a 100Hz sine wave causing a peak deviation of 550 Hz. The unmodulated carrier wave amplitude is 5 volts. Determine the amplitude and relative phase of each component of the frequency spectrum.

2. Estimate the bandwidth of the FM signal in question 1 using the Carson's rule and the modified Carson's rule. Compare these bandwidths with the actual spectrum of the signal found in question 1. What is the maximum size of any components outside the Carson bandwidth? What fraction of the total power of the FM signal is contained within the Carson bandwidth?

3. Draw typical time waveforms and spectra for AM, DSB and FM modulation when the modulating signal is a tone of frequency fm. Hence compare the bandwidth usage and efficiency of AM, FM and DSB.

4. Sketch the block diagram for a circuit to produce the composite modulating signal used in a standard, stereo FM broadcast transmitter.

5. The bandwidth assigned to commercial FM broadcast stations is 88 to 108MHz. Estimate the maximum number of stereo FM stations which could operate within this bandwidth. The maximum allowed frequency deviation of broadcast transmitters is 75kHz. Why is it unlikely in practice that this number of stations could share this bandwidth?

6. An FM signal is being modulated with a tone signal of frequency fm and amplitude Am.

    a) Determine the values of the modulating index β for which the carrier component of the FM spectrum will disappear.
    b) In a certain experiment with fm = 1kHz and increasing Am (starting from zero volts), the carrier component of the FM spectrum is reduced to zero for the first time when Am = 2 volts. What is the frequency sensitivity (k_f) of the FM transmitter? What would be the value of Am at which the carrier component disappears for the second time?

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For data communications, the transmitter portion of a modem has a carrier frequency of 1500 Hz. It is frequency modulated by a 100 Hz sine wave causing the carrier to reach a peak deviation of ± 550 Hz. The unmodulated carrier amplitude is 5.0 volts. Determine the amplitude and relative phase of each component of the frequency spectrum.

Solution:

The modulation index is:

$$\frac{\text{peak deviation}}{\text{modulating freq.}} = \frac{550\text{ Hz}}{100\text{ Hz}} = 5.5$$

This value is used to find the Bessel coefficients for each of the significant components.

Next, the amplitudes and relative phase angles of each frequency component are calculated. Remember that all of the upper sidebands *normally* start at zero phase angle as do the even ones on the lower side. The odd, lower sidebands *normally* start at a relative angle of 180°. The Bessel coefficients that have negative values will cause the phases of the corresponding sidebands to change by an additional 180°. In the following chart, which summarizes all the results, the (+) sign is used for zero phase angle and the (−) is used for the 180° starting phase.

Frequency	*Bessel Coefficient*	*Normal Phase*	*Final Amplitude and Phase*
600 Hz	$J_9 = +0.02$	−	−0.10 volts
700	$J_8 = +0.04$	+	+0.20
800	$J_7 = +0.10$	−	−0.50
900	$J_6 = +0.18$	+	+0.90
1000	$J_5 = +0.32$	−	−1.60
1100	$J_4 = +0.39$	+	+1.95
1200	$J_3 = +0.27$	−	−1.35
1300	$J_2 = -0.14$	+	−0.70
1400	$J_1 = -0.34$	−	+1.70
1500	$J_0 = 0.00$	+	no carrier component
1600	$J_1 = -0.34$	+	−1.70
1700	$J_2 = -0.14$	+	−0.70
1800	$J_3 = +0.27$	+	+1.35
1900	$J_4 = +0.39$	+	+1.95
2000	$J_5 = +0.32$	+	+1.60
2100	$J_6 = +0.18$	+	+0.90
2200	$J_7 = +0.10$	+	+0.50
2300	$J_8 = +0.04$	+	+0.20
2400	$J_9 = +0.02$	+	+0.10

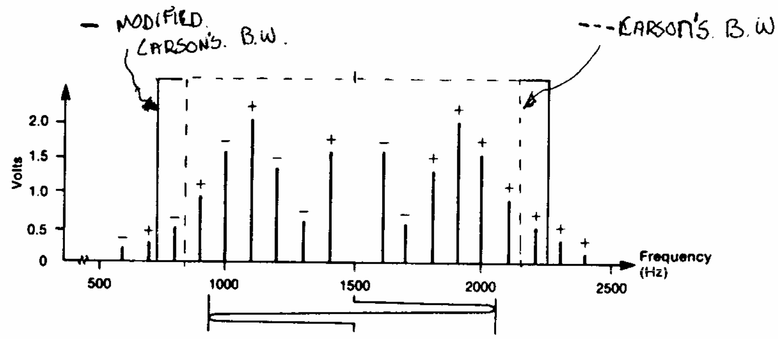

Figure 3.1 Sidebands for a 1500 Hz carrier deviated ± 550 Hz by a 100 Hz sine wave.

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2). The bandwidth is given by Carsons rule to be.

$$B.W. = 2(D+1)W.$$

where W is the maximum audio frequency and D is the modulating index (β) when $f_m = W$.

Thus here.

$$W = 100 \text{ Hz}.$$

&

$$D = \frac{\Delta}{W} = \frac{550}{100} = 5.5$$

Thus. $B.W. = 2(5.5+1) \times 100.$

$= 1300$ Hz

The Modified Carson Rule gives.

$$B.W = 2(D+2)W$$

$= 1500$ Hz.

[See Fig. 3.1 for sketch].

We see by plotting the calculated B.W's of this system on the actual spectrum that some components are outside the predicted bandwidths.

Components outside the Carson bandwidth are as large as 10% the ~~carrier~~ unmodulated carrier magnitude.

Components outside the modified Carsons bandwidth are smaller; < 5% the unmodulated carrier magnitude.

The total power transmitted is just the power of the unmodulated carrier.

$$\text{TOTAL Power} = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$= (5)^2 = 25 \text{ WATTS.}$$

The power transmitted in the Carson BW is just the sum of the component powers in this range.

ie. Power in Carsons B.W. $= 2\left[(1.7)^2 + (0.7)^2 + (1.35)^2 + (1.95)^2 + (1.6)^2 + (0.9)^2\right] + (0)^2$

$\Rightarrow$ Power in BW $=$ 24.75 WATTS.

Fraction of total power in Carsons B.W.

is. $\dfrac{\text{Power in Carson BW}}{\text{Total Power}} = \dfrac{24.75}{25}$

$= 99\%$

Thus most of the signal power is transmitted in this B.W.

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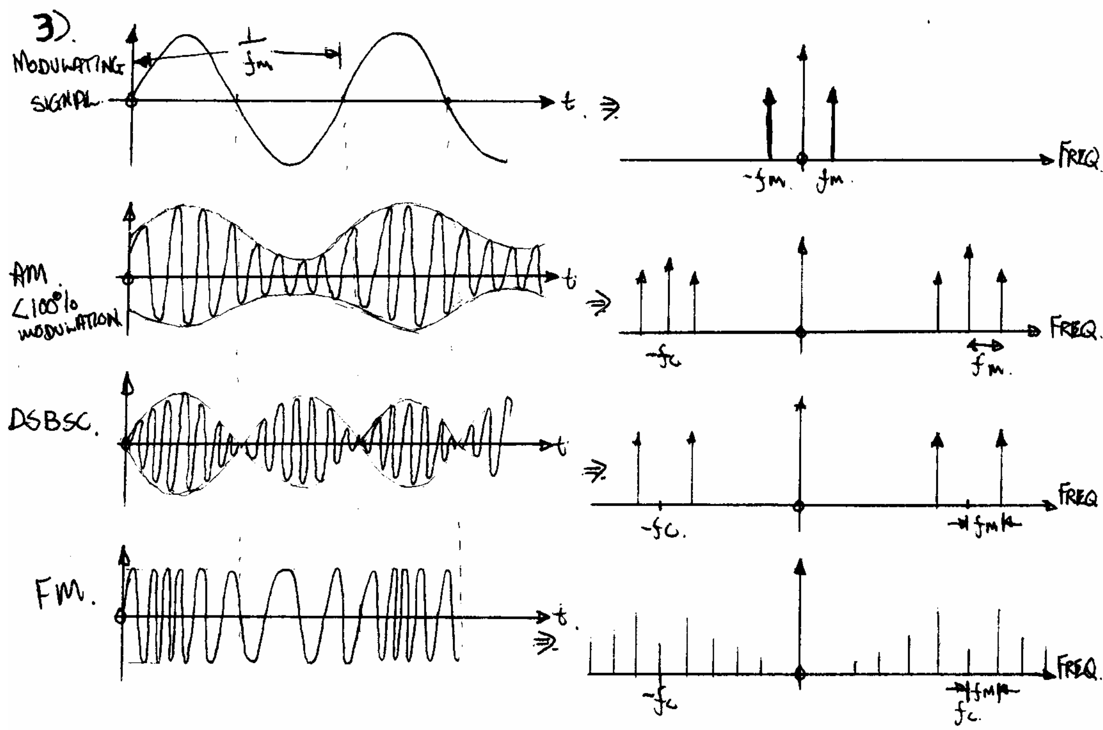
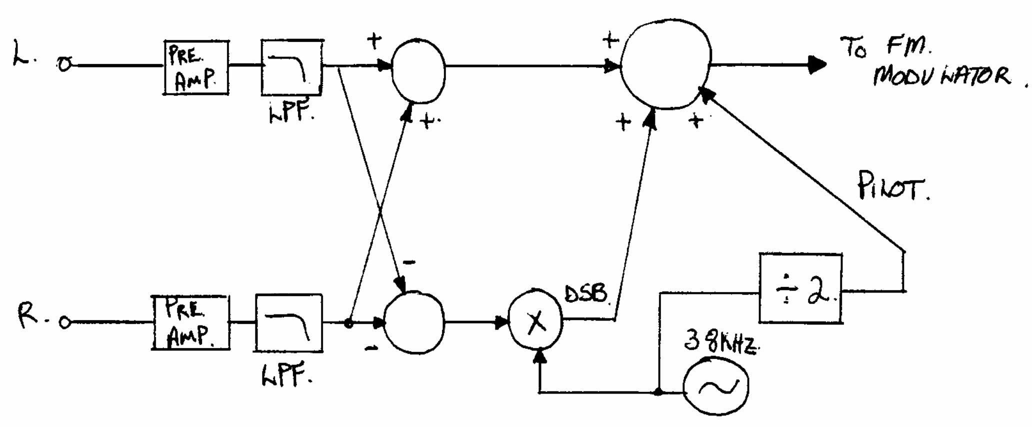
3).



4).

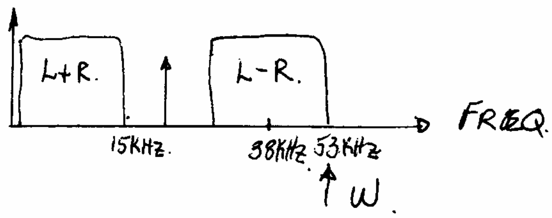


5.

Useable B.W = 108 − 88 = 20 MHz.

Maximum Frequency of stereo audio signal is 53 KHz.

Since we have.



Thus. $B_{(MIN)} = \frac{Am.\,fd}{W.}$

$$= \frac{1.75\,KHz}{53\,KHz} = 1.415.$$

Thus using the modified Carsons rule the BW of one station would be.

$$B.W_{(1)} = 2(D+2)\,W.$$

$$\Rightarrow \quad B.W_{(1)} = 2(1.415+2).\,53\,KHz.$$

$$= 362.\,KHz.$$

Thus. maximum number of stations in given B.W. is.

$$\text{NUMBER OF STATIONS} = \frac{\text{TOTAL B.W}}{B.W\,(1)} = \frac{20\times10^{6}\ Hz}{362\times10^{3}\ Hz}.$$

$$\doteq 55 \text{ stations.}$$

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a) The spectrum of the FM signal will have components at the carrier frequency and at integer multiples of the modulating frequency (f_m) above and below the carrier. The carrier component is described by

$$A_c J_0(\beta) \cos \omega_c t.$$

Thus the carrier component will be zero when $J_0(\beta) = 0$ ie at the roots of the Bessel functions of the first kind, order 0.
These roots are given in the Electrical Engineering Tables. From the tables we see $J_0(\beta) = 0$ when

$$\beta = 2.4048,\ 5.5201,\ 8.6537,\ 11.7915 \text{ etc}$$

b) The modulation index β is defined to be

$$\beta = \frac{A_m f_d}{f_m}$$

In this experiment $f_m = 1$ kHz.
The carrier component will disappear for the first time when $\beta = 2.4048$.
If the amplitude of the modulating signal (A_m) is 2 volts at this point then

$$\beta = \frac{2.0 \times fd}{1 \times 10^3} = 2.4048$$

Thus the frequency sensitivity of the FM modulator fd must be

$$fd = \frac{2.4048 \times 10^3}{2.0}$$

$$\underline{fd = 1.2024 \text{ (kHz/volt)}}$$

The carrier component dissappears for the second time when $\beta = 5.5201$.

Thus $$\beta = \frac{Am\ fd}{fm} = 5.5201$$

or $$\underline{Am = \frac{5.5201 \times fm}{fd} = 4.591 \text{ volts}}$$

Note; this experiment is a very accurate technique for measuring the frequency sensitivity of an FM system.